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SUMMARY

A study to develop procedures for applying decoupling theory in the control of a long, flexible beam in low Earth orbit was performed. Control involved commanding changes in pitch attitude as well as nulling initial disturbances in the pitch and flexible modes. Control-force requirements as well as the effects of parameter uncertainties on the decoupling process were analyzed.

Two methods for developing decoupled control techniques were investigated. For the first method, the system was completely decoupled (that is, the number of control actuators equaled the number of modes in the model). Certain actuators were then eliminated, one by one, which resulted in some or all modes not fully controlled. For the second method, specified modes of the system were excluded from the decoupling control law by employing fewer control actuators than modes in the model. Finally, in both methods, adjustments were made in the feedback gains to include the uncontrolled modes in the overall control of the system.

Both methods produced initial effects on the system response characteristics, but these effects were generally small and were damped out in relatively short time periods. The second method was superior in that it employed a simple gain-adjustment procedure. However, the first was effective in producing a single actuator system.

The control-force requirements were found to depend on three factors: (1) the number and location of control actuators, (2) the number of structural modes considered in the model, and (3) the values selected for the closed-loop dynamics of the system. Of these factors, the value selected for the closed-loop pitch angular frequency was by far the most significant. Over a range of 0.1 to 1.0 rad/sec, the control-force requirements increased by four to five orders of magnitude.

Decoupled control was found to be essentially insensitive to uncertainties in the model parameters. This characteristic was important in establishing the simple gain-adjustment procedure needed for overall control of the system.

INTRODUCTION

A number of future space applications will involve the deployment of very large structures in Earth orbit (refs. 1 and 2). Because of weight limitations, these structures will necessarily be extremely thin and flexible and will require the development of new control technologies. In the last several years, various methods of controlling the attitude and modal displacements of large space structures have been developed and analyzed (ref. 3); however, much work remains to be done before simple, reliable control procedures can be established.

Decoupling theory (for example, ref. 4) is a convenient tool for devising control laws for structures with a large number of state variables because it allows independent control of each state. For example, the pitch attitude can be controlled so that the flexible modes are unaffected. The theory also permits the feedback gains to be adjusted in order to achieve desired closed-loop dynamics without destroying the independent control capabilities. Several decoupling techniques have been studied recently (for example, refs. 5 to 7); however, the theory has not been fully utilized because of a basic limitation wherein complete modal decoupling requires the number of control actuators to equal the number of modes in the model. Complete decoupled control is usually not achievable in practical application because a large space structure may have an infinite number of flexible modes; hence, procedures must be developed which maintain control of the structure with a small number of control actuators.

This study presents techniques which use decoupling theory and state-variable feedback to control the pitch attitude and modal amplitudes of a long, flexible beam in orbit. Structures similar to a long beam could have direct space application because of the relatively simple one-dimensional control problem. In addition, knowledge gained from the study of the beam could be applied to the control of more complex space structures. The computer program described in reference 4 was used for the decoupling calculations.

In the present study, some approximations are incorporated into the decoupling procedure to provide adequate control of all modes. The approximations involve adjustments in the control-influence coefficients and in the feedback gains which produce simplified procedures for achieving overall control of the system. It should be noted that the results of this study represent the ideal performance of the decoupled control approach. Measurements are assumed to be provided for the uncontrolled modes; hence no observation spillover is present, and stability can be guaranteed for the system. Control spillover, however, is present because the number of control actuators is less than the number of modes in the model. The control spillover effects degrade performance, but stable systems can always be achieved with proper selection of feedback gains. The important problem of stability in the presence of both control and observation spillover due to the effects of uncontrolled modes (for example, refs. 8 to 10) is not included in this analysis.

Results are presented which show the various techniques employed in determining the required feedback gains and their effects on the uncontrolled modes. These results are depicted primarily as time histories of modal responses and control-force requirements for step commands on the system. Control-force requirements are presented in relation to the closed-loop dynamics which cover a wide range of frequencies and damping ratios. Effects of inaccurate knowledge of the control-influence coefficients, which lead to errors in the calculated feedback gains, are included in the analysis.

SYMBOLS

A	$= \left[\begin{array}{c c} 0 & I \\ \hline W & 0 \end{array} \right]$ (see eq. (6))
A_n	modal amplitude of nth generic mode, m
\bar{A}'	column vector of modal amplitudes
B	control-influence matrix
\hat{B}	$= \left[\begin{array}{c} 0 \\ \hline B \end{array} \right]$ (see eq. (8))
BF	matrix product of B and F
C	matrix relating output (decoupled) vector to state vector
D, K, P, Q, R, S, T, U	constants
E_n	effect of external forces on the nth generic mode
F, F', F''	feedback gain matrices (see appendix)
$F_{i,j}$	element in feedback gain matrix (for example, $F_{1,8}$ is gain value in first row and eighth column of F)
f	control force
\bar{f}	m-vector of thrust control forces
G	feedforward gain matrix
J	pitch-axis moment of inertia, kg-m^2
l	undeformed length of beam, m
M_n	generalized mass of beam in nth mode, kg
M, N	auxiliary matrices used in appendix
m	number of control actuators
n	number of flexible modes plus pitch mode
T_p	external pitch torque
t	time, sec
\bar{r}	column vector of beam deflections

s	Laplace operator
v	input command
\bar{v}	m-vector of command inputs
W	diagonal matrix of undamped eigenvalues squared
x	state variable
\bar{x}	2n-vector of state variables
y	output variable
\bar{y}	m-vector of state variables to be controlled in decoupled manner
ζ	damping ratio
ζ_n	augmented damping ratio of nth generic mode
ζ_θ	augmented damping ratio of pitch mode
θ	pitch angle between undeformed longitudinal axis and local vertical, rad
ξ	position along beam
ρ	beam mass per unit length, kg/m
ϕ	modal shape matrix
ϕ_z^n	zth component of modal shape function corresponding to nth mode
ω	angular frequency
ω_c	orbital angular velocity (orbital frequency), rad/sec
ω_n	angular frequency of nth generic mode (augmented or natural), rad/sec
ω_θ	augmented pitch angular frequency, rad/sec
Subscript:	
u	unaugmented

A dot over a symbol denotes differentiation with respect to time.

METHOD OF ANALYSIS

Equations of Motion

Equations of motion for a thin homogeneous uniform beam whose center of mass is assumed to follow a circular orbit have been developed in reference 11. When all rotations and transverse elastic displacements are assumed to occur within the plane of the orbit, and when the Earth's gravitational field is considered to be spherically symmetric, these equations can be reduced to

$$\frac{d^2\theta}{dt^2} + \frac{3\omega_c^2 \sin 2\theta}{2} = \frac{T_p}{J} \quad (1)$$

$$\frac{d^2A_n}{dt^2} + \left[\omega_n^2 - \omega_c^2(3 \sin^2 \theta - 1) - \left(\frac{d\theta}{dt} - \omega_c \right)^2 \right] A_n = \frac{E_n}{M_n} \quad (2)$$

For the development of the actuator modeling and subsequent decoupling analysis, equations (1) and (2) can be linearized (ref. 12) about a position with zero structural deformation and alignment of the beam along the local vertical. It is assumed that all elastic displacements are small in comparison to the beam length. The resulting linearized system equations are

$$\frac{d^2\theta}{dt^2} + 3\omega_c^2\theta = \frac{T_p}{J} \quad (3)$$

$$\frac{d^2A_n}{dt^2} + \omega_n^2 A_n = \frac{E_n}{M_n} \quad (n = 1, 2, \dots) \quad (4)$$

where structural damping is assumed to be zero.

Equations in State-Vector Form

Equations (3) and (4) can be written in state-vector form by defining

$$\left. \begin{array}{ll} \theta = x_1 & \frac{d\theta}{dt} = \dot{x}_1 = x_{n+1} \\ A_1 = x_2 & \frac{dA_1}{dt} = \dot{x}_2 = x_{n+2} \\ \vdots & \vdots \\ \vdots & \vdots \\ A_{n-1} = x_n & \frac{dA_{n-1}}{dt} = \dot{x}_n = x_{2n} \end{array} \right\} \quad \text{and} \quad \quad (5)$$

so that

$$\dot{\bar{x}} = \begin{bmatrix} 0 & | & I \\ \hline W & | & 0 \end{bmatrix} \bar{x} + B_c \bar{u}_c \quad (6)$$

where

$$\bar{x} = [x_1, x_2, \dots, x_n, x_{n+1}, \dots, x_{2n}]^T \text{ is the state vector}$$

$0 = n$ by n null matrix

$I = n$ by n identity matrix

$$W = \begin{bmatrix} -3(\omega_c)^2 & & & 0 \\ & -(\omega_1)^2 & & \\ & & \ddots & \\ 0 & & & -(\omega_{n-1})^2 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 \\ \hline I \end{bmatrix}$$

and

$$\bar{u}_c = [T_p/J, E_1/M_1, \dots, E_{n-1}/M_{n-1}]^T$$

is the control vector.

Modeling of Controls

It is assumed that m thrust actuators are located along the beam at points $\xi_1, \xi_2, \dots, \xi_m$, where ξ lies along the beam's undeformed longitudinal axis and $\xi = 0$ corresponds to the mass center of the undeformed beam. The actual control forces associated with these actuators are designated f_1, f_2, \dots, f_m , respectively. For small elastic displacements, the component of the control force parallel to the ξ -axis is considered negligible. Equations for the pitch control force and the modal control forces are developed in reference 11, so that the control vector becomes

$$\bar{u}_c = \begin{bmatrix} -\xi_1/J & -\xi_2/J & \dots & -\xi_m/J \\ \phi_z^1(\xi_1)/M_1 & \phi_z^1(\xi_2)/M_1 & \dots & \phi_z^1(\xi_m)/M_1 \\ \vdots & \vdots & \ddots & \vdots \\ \phi_z^{n-1}(\xi_1)/M_{n-1} & \dots & \phi_z^{n-1}(\xi_m)/M_{n-1} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \quad (7)$$

If the n by m matrix appearing in equation (7) is denoted as B , equation (6) may be expanded in the following form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{2n} \end{bmatrix} = \begin{bmatrix} & & & & & & I \\ & 0 & & & & & \\ \vdots & \vdots & \ddots & \vdots & & & \\ -3(\omega_c)^2 & & & 0 & & & \\ & -(\omega_1)^2 & & & & & 0 \\ & & \ddots & & & & \\ & & & -(\omega_{n-1})^2 & & & \\ 0 & & & & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ B \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \quad (8)$$

where B is the control-influence matrix and the f values are the control thrust forces. The generalized masses appearing in equation (7) can be evaluated for homogeneous free-free beams and shown to be independent of the mode number (ref. 13). Specifically, $M_j = \rho l$.

Decoupling Procedure

The computer program described in reference 4 was used for the decoupling calculations. The system equations are

$$\dot{\bar{x}} = A\bar{x} + \hat{B}\bar{f} \quad (9)$$

$$\bar{y} = C\bar{x} \quad (10)$$

where equation (9) is equation (8) written in abbreviated form and equation (10) is the output equation with \bar{y} representing the states to be decoupled. The decoupling control law is taken as

$$\bar{f} = F\bar{x} + G\bar{v}$$

where \bar{v} is the input command vector, and F and G are the feedback and feedforward gain matrices, respectively. The output is related to the input through the transfer function $H(s)$, or

$$\hat{\bar{y}}(s) = H(s) \hat{\bar{v}}(s) \quad (11)$$

where \bar{y} and \bar{v} are required to be of the same order. The circumflex in equation (11) indicates the Laplace transform and

$$H(s) = C(sI - A - BF)^{-1}BG$$

The computer program determines the F and G matrices so that the transfer function is diagonal and nonsingular, thus providing independent control for each of the decoupled (output) variables. A schematic diagram of the decoupled control system is shown in figure 1. An example presented in the appendix illustrates the procedure for decoupling the rigid-body (pitch) mode and the first flexible mode for a four-mode model of the beam. The second and third flexible modes are uncontrolled in this example. These uncontrolled modes will be influenced by effects due to control of the decoupled modes. The purpose of the present study is to determine methods for reducing these undesired effects.

General Considerations

The results of the present study pertain to a modal control analysis for a long (100-m) flexible free-free beam in a 250-nautical-mile circular orbit, where $\omega_c \approx 10^{-3}$ rad/sec. The beam is represented by a long, slender, hollow tube made of wrought aluminum with an outside diameter of 10.79 cm and thickness of 1.06 cm. The mass of the beam is 1000 kg, and the pitch-axis moment of inertia is 840 000 kg-m². The analysis includes models with up to five flexible modes. A continuous-beam frequency analysis for this model gives the first through the fifth modal natural frequencies as 0.0566, 0.1697, 0.3051, 0.5053, and 0.80 rad/sec, respectively. In units of Hertz, these values are 0.0090, 0.0270, 0.0486, 0.0805, and 0.1274, respectively.

A modal control system requires measurements of the modal responses (amplitudes and rates) for the feedback control. Various methods are available but all involve measuring physical quantities and processing the measurements to establish the modal responses. A simple scheme for determining the modal amplitudes can be implemented with p sensors, where p is the number of flexible modes in the model. If the sensors are located to measure p discrete displacements r and if all p modes contribute to the measurements, then the transformation

$$\bar{A}' = \phi^{-1}\bar{r}$$

provides the desired modal amplitude data \bar{A}' . The modal rates can be derived from the amplitude data by differentiation.

Decoupled Control Techniques

Two techniques are used in the decoupled control analysis. In the first approach, the model is completely decoupled; that is, the number of control actuators equals the number of modes in the model, and full control of all the modal responses is achieved. As previously stated, the number of modes includes the rigid-body (pitch) mode plus the structural (flexible) modes. Then, various actuators are eliminated from the control model, and several techniques are employed to adjust the original feedback gains to obtain adequate control of the system with the reduced number of actuators.

The second approach incorporates a reduced number of control actuators into the control process; only specified modes are decoupled. The modes remaining in the model are uncontrolled and, hence, are affected by control of the decoupled modes. Several techniques are used to adjust the feedback gains to obtain adequate control of the complete model.

Presentation of Data

The results of the control analysis are presented primarily as time histories of modal amplitudes and control requirements for selected values of the closed-loop dynamic characteristics of the beam. These time histories illustrate the potential use of the decoupling techniques for achieving control of all modes. Time-history data are also presented which show the effect of parameter uncertainty on the decoupling process as well as on the uncontrolled modes. In addition, parametric data plots of control-force requirements and of uncontrolled mode effects are shown for a range of selected closed-loop dynamics.

DISCUSSION OF RESULTS

The results are discussed for the two decoupling techniques. The first approach starts with a completely decoupled four-mode model and proceeds to reduce the number of control actuators. The original feedback gains for the four-control completely decoupled model are adjusted so that one, two, or three actuators are sufficient to control the model. In the second approach, beam models with up to five flexible modes are decoupled by using fewer actuators than modes in the model. Except for a special case in which an actuator is located exactly at each end of the beam, this procedure leaves some of the modes uncontrolled. Information derived from this special case allows the feedback gains for various control models to be adjusted to obtain control for all modes.

The effect of the various control techniques used in the analysis is illustrated by time histories of the modal responses and control-force requirements. The time histories are produced by instantaneous step-control commands which are input either as commands for pitch-angle attitude change or as zero commands designed to reduce all initial displacements to zero. In all cases, the pitch command value is taken as 0.01 rad. In the same manner, the initial

displacements for the pitch and modal amplitudes are taken as 0.01 rad and 0.01 m, respectively. The resulting initial values for the control forces are indicated on the time histories by arrows.

Procedure Derived From Complete Decoupling

An example of time-history responses for a completely decoupled four-mode model is shown in figure 2. The feedback gains employed for this case are presented in table I as F_1 . The gains correspond to certain selected values of the closed-loop dynamics ω and ζ as indicated in figure 2. As shown in the appendix, second-order dynamics ($s^2 + 2\zeta\omega s + \omega^2$) are required in calculating the gains from the ω and ζ values. The A matrix for this model is given in the appendix. The control-influence matrix is given in table II as B_1 and corresponds to the arrangement of the control actuators depicted in figure 2.

In all cases throughout this paper, the feedforward gains G are not presented. These gains operate on the command inputs to produce the decoupled output and are merely the inverse of the B matrix (see appendix). Each G column can be multiplied by a constant to change the output sensitivity.

Figure 2(a) shows that all modes can be completely decoupled if the number of control actuators equals the number of modes. In addition, the feedback gains completely damp out all initial disturbances for a zero command (fig. 2(b)). (A zero command corresponds to a zero input in all v command quantities.)

The control-force requirements for the completely decoupled model are shown in figure 3 for a range of ω_0 values and $\zeta_0 = 0.5$. Values of ω and ζ for the flexible modes are not pertinent for the case of a pitch command because the flexible responses remain at zero. As expected, figure 3 shows a large drop in control-force requirements in the lower frequency range, inasmuch as this range represents a system with low stiffness.

Elimination of one control actuator.— Figure 4 illustrates the effect of taking the completely decoupled model and eliminating the control actuator in the middle of the beam by setting the third row of F_1 to zero. (The third row of the G matrix is also set to zero.) Figure 4(a) clearly shows that the first and third modes cannot be controlled without some adjustments in the feedback gains. After some random trial-and-error adjustments, the time-history responses shown in figures 4(b) and 4(c) were obtained. These responses correspond to increasing only the $F_{1,8}$ gain by a factor of 10. As shown in figure 4(b), the flexible mode responses are not initially decoupled from the pitch response, but after about 100 sec, they essentially damp out. Similar results are shown for the zero command in figure 4(c).

Figure 5 is similar to figure 4, except that a different set of closed-loop dynamics was used to calculate the feedback gains. This procedure was used in an effort to improve the results of figures 4(b) and 4(c), where the initial coupling effects on the modal responses are relatively large. The $F_{1,8}$ gain value for figures 5(b) and 5(c) was increased by a factor of 200. Comparison

of figures 4(c) and 5(c) shows that for a zero command, the magnitudes of the coupling effects are about the same; however, the responses in figure 5(c) require over 400 sec to damp out. For a pitch command, comparison of figures 4(b) and 5(b) shows smaller coupling effects in the latter. The ideal solution would be to use the gains of figure 5 for pitch commands and the gains of figure 4 for zero commands in order to obtain the faster response.

Elimination of two control actuators.- Figure 6 illustrates the example in which the middle and right-end control actuators are eliminated from the completely decoupled model. In this case, the third and fourth rows of the F_1 and G matrices are set to zero. As shown in figure 6(a), none of the modes are controlled. To yield the results shown in figures 6(b) to 6(e), more elements of the F matrix required adjusting than in the case where only one control actuator was eliminated. For figures 6(b) and 6(c), the sign of $F_{1,2}$ was changed, and $F_{1,8}$ was doubled; these adjustments resulted in adequate pitch- and zero-command control. For the results shown in figures 6(d) and 6(e), the $F_{1,8}$ gain value was again doubled; this readjustment resulted in smoother responses in A_3 and f_1 , but provided no improvement in the coupled effects. In fact, the zero command produced a 20-percent increase in the effect on A_1 .

Elimination of three control actuators.- In figure 7 all control actuators, except the one at the left end, are eliminated from the completely decoupled model. In order to achieve control of all modes, a large number of adjustments in the F matrix gains were required. For the pitch and zero commands shown in figures 7(b) and 7(c), the adjustment of these gain values was based on data from a special two-actuator case (see F_2 , table I) which is described in a later section. The signs of $F_{1,3}$ and $F_{1,4}$ were made to correspond with those of the special case; and the ratios of the $F_{1,5}, \dots, F_{1,8}$ gains to the $F_{1,1}, \dots, F_{1,4}$ gains (i.e., $F_{1,5}:F_{1,1}$, etc.) were made to equal the ratios of the corresponding values of the special case.

Summary of results.- The foregoing results indicate that a completely decoupled control analysis can be used to develop a control model with fewer actuators than modes in the model. However, the random method of adjusting the gains for reasonable control is difficult to use and requires many trial-and-error attempts. A more practical solution to the control problem is discussed in the following section.

Procedure With Incomplete Decoupling

An example of time-history responses for a four-mode model decoupled with only two control actuators is shown in figure 8. (The beam model consists of the pitch mode plus the first three flexible modes; the A matrix is given in the appendix.) This case is referred to as a special case of incomplete decoupling because of the unique location of the actuators. The control-influence matrix B_2 is given in table II and corresponds to an actuator located exactly at each end of the beam. In this case, the pitch and first flexible modes are decoupled. The feedback gains, presented in table I as F_2 , correspond to certain selected values of the closed-loop dynamics ω and ζ for the decoupled modes, as indicated in figure 8. Even though the second and

third flexible modes are not included in the decoupling control law, figures 8(a) and 8(b) show that these modes are controlled, both for a pitch command and for a zero command.

Special case for determining feedback.- In the special case, where the control actuators are located exactly at the ends of the beam, the magnitudes of the elements in the rows for the flexible mode coefficients in B_2 are the same. Because of the unique interaction of the actuators with all the flexible modes, this special case results in a full order F matrix (that is, no zero columns), thus providing control for all modes. This F matrix is derived by the computer program described in reference 4 and has the form given in table III as F''_{exact} . Hereafter, the B_2 matrix for this special case is referred to as the "exact" B matrix. The F''_{offset} matrix shown in table III corresponds to cases where the control actuators are offset; that is, they are not located exactly at the ends of the beam, or where the actuators may be exactly at the ends, but the beam possesses slightly different structural characteristics at each end. (The B matrix for this case is shown in table II as B_3 .) As shown for F''_{offset} in table III, the zero columns provide no control for the second and third flexible modes. Similar results are obtained when deviations occur in any two rows of B_2 . This particular case will be discussed in a subsequent section.

In table III, only the first four columns are shown for F''_{exact} ; the last four columns are derived by replacing S with R , and Q with P . The angular frequencies in F''_{exact} are the open loop (unaugmented) values for the beam. Also, the F'' matrices shown in the table include only that part required to change the dynamics; the complete form of F is indicated in the appendix.

As noted from table III, the F''_{exact} matrix requires fourth-order dynamics to calculate the gains. Values selected for ω_0 , ω_1 , ζ_0 , and ζ_1 are used to determine the terms in the equation shown below the matrix. As indicated, these terms are used as multiplying factors in the various columns of the F''_{exact} matrix.

Parameter uncertainty.- The effect of parameter uncertainty was investigated by incorporating errors of ± 10 and ± 20 percent, randomly, in the B matrix. A typical result, using B_4 from table II, is compared in figure 8(c) to the no error case. The original feedback gains F_2 were used for the case with errors.

The comparison shown in figure 8(c) indicates a negligible effect of parameter uncertainty on the decoupling control process. Comparison of zero command responses in figures 8(b) and 8(d) gives the same result. Figures 8(e) and 8(f) illustrate examples of combined commands with parameter uncertainty; that is, a 0.01-rad pitch change is required along with zero commands to reduce initial displacements in the three modal amplitudes. The results show that adequate control is maintained despite parameter uncertainty.

The uncertainty in beam model parameters would generally have little or no effect on the decoupling process because the unaugmented values of ω_0 and ω_1 are much smaller than those selected for the closed-loop values in the present analysis.

Summary of parameter uncertainty effects.- The fact that the effect of control-influence-parameter uncertainty is small can be important for two reasons. First, inaccuracies which may be present in any actual control-system design will not produce undesirable effects on the decoupling process. Second, this fact can be used in manipulating control models (B matrix) in order to obtain full-order feedback gain matrices. For example, the B matrices of an actual control model and an exact model, if similar, can be interchanged in the calculation of feedback gains with negligible effects on the decoupling process. This is discussed in a subsequent section.

Effect of decoupling on uncontrolled mode.- As shown in figure 8(a), the decoupling process has an effect on the second flexible mode, which is one of the uncontrolled modes. In the case depicted, this effect is small and damps out after a period of time. The effects on the second mode are summarized in figure 9 for a range of ω_0 and ζ_0 values. As indicated in the figure, the closed-loop value selected for ω_0 has, by far, the largest effect on the second-modal amplitude. The amplitude is relatively insensitive to changes in the closed-loop damping term ζ_0 .

Control-force requirements.- The control requirements for incompletely decoupled models are presented in figures 10 and 11 and in table IV. Most of the results apply to the model with pitch and three flexible modes. The trend shown for pitch control for the incompletely decoupled models in figure 10 is similar to that shown in figure 3 for completely decoupled models; however, the requirements increase more rapidly at the larger values of ω_0 . Data for the higher order beam models are included in figure 10 because the modal response characteristics of these models are analyzed in subsequent sections.

Examples of zero-command control requirements are shown in figure 11. For the case in the lower plot, the value of 0.5 rad/sec selected for the closed-loop value of ω_0 results in a dominant pitch mode feedback; that is, the feedback gains in the first column of F are much larger than the other gains. As indicated by the figure, this condition requires much larger control forces than those in the upper plot, where $\omega_0 = 0.1$ rad/sec.

Control-force requirements are compared in table IV for pitch and zero commands. The data are similar in the three cases shown. Inasmuch as each involves a change of 0.01 rad in pitch, this similarity indicates that relatively small control forces are required for the flexible-mode zero commands.

Decoupling by ratio method.- As previously indicated, incomplete decoupling results in some modes of the model remaining uncontrolled. All modes are controlled in the incomplete decoupling process only in the special case where the magnitudes of the rows for the flexible mode coefficients in the B matrix are the same. A case in which the magnitudes are slightly different is shown in

figure 12 for the example in the appendix. The B matrix is given in table II as B_3 . The feedback gain matrix for this case (for selected values of ω and ζ given in fig. 13) would be the same as F_3 in table I, except with zeros in the third, fourth, seventh, and eighth columns.

The time histories in figure 12(a) illustrate the effects on the uncontrolled modes caused by the zero gains in the F matrix. The results in figures 12(b) and 12(c) were obtained by using a ratio method to adjust the zero-gain columns. These columns were determined from the ratios of the full-order F_2 gain matrix corresponding to the special case previously discussed. In applying the ratio method, the third column of F_3 was adjusted so that its ratio with the first column in F_3 was the same as that of corresponding columns in F_2 . The same procedure was used with respect to the fourth and second columns, the seventh and fifth columns, and the eighth and sixth columns. The time histories in figures 12(b) and 12(c) indicate that the adjusted gains provide sufficient control for all modes, both for pitch commands and zero commands. There are small initial effects, but they damp out after about 200 sec.

Summary of effect of ratio method.— Results in figures 12 and 8 show that the case for the ratio method compares favorably with the special case. Even though the feedback gains are entirely different in each case, the overall effects of controlling the system are essentially the same. The pitch-command response results in figure 12(b) are similar to those shown in figure 8(a) for the special case where the only adverse effect occurs in A_2 . The control-force requirements, however, are about three times larger in figure 12(b). The zero-command results are also similar for the two cases (figs. 12(c) and 8(b)), except for more rapid initial oscillations in A_1 and A_3 in figure 12(c).

Extension of ratio method.— It has been shown that control of all modes with an incompletely decoupled system can be accomplished by a simple method of gain adjustment. This ratio method was shown for a two-actuator, four-mode case where the values of the actual (offset) and "exact" control-influence coefficients were nearly the same. The following sections of the report will discuss the extension of the ratio method to models with a larger number of structural modes, as well as to higher order control-actuator models. In addition, the ratio method can be applied to cases where the actual and exact control-influence coefficients are noticeably different. An example of such a case is presented in the following section.

Change in control-actuator location.— The results in figure 13 pertain to the four-mode model where the f_2 control-actuator location has been changed as shown. As in the case of figure 12, the pitch and first flexible modes are decoupled. The actual control-influence matrix used is given in table II as B_5 . The feedback gain matrix calculated for this case would be the same as that for F_4 (see table I), except with zeros in the third, fourth, seventh, and eighth columns, and would lead to uncontrolled oscillations in the second and third flexible modes similar to the oscillations shown in figure 12(a). The full-order F_4 matrix was determined by use of the ratio method, as previously explained.

Time histories of pitch command and zero command, calculated with the adjusted feedback gains, are shown in figures 13(a) and 13(b), respectively. Adequate control is attained in all modes; however, well over 300 sec are required to damp out the initial effects, with the largest effect occurring in A_2 . The adjusted gain matrix F_4 required a change in sign in the third, seventh, and eighth columns due to the sign arrangement in B_5 .

For the time histories in figures 13(c) and 13(d), an alternate method of determining the feedback gain matrix was employed. F_5 was calculated according to

$$F_5 = B^{-1}[BF]'$$

where B^{-1} is the inverse of the first two rows of the B_5 matrix and $[BF]'$ corresponds to the first two rows of the BF matrix determined for the special case. This BF matrix was used because it was the result of fourth-order dynamics in F and produced control in all modes. As in the case just discussed, the signs in the same three columns of F_5 were changed. The F_4 and F_5 matrices are similar in that the ratios between every other column are the same for corresponding columns in the two matrices; both matrices are based on the F_2 matrix. The gain magnitudes are different for the two matrices because second-order dynamics were originally involved in calculating F_4 .

Comparison of the two sets of data (figs. 13(a) and 13(b) versus 13(c) and 13(d)) shows similar results. However, the alternate method produces more favorable response characteristics, both in the initial effects and in the time to damp to steady state. The ratio method produces control forces for a pitch command which are three times as high and forces for a zero command which are one-third as high as those required by the alternate method.

Summary of control-actuator-location effect.- The control-actuator arrangement shown in figure 13 is a poor selection. Comparison with figure 12 illustrates the significance of actuator location in two regards. First, the arrangement shown in figure 13 required more manipulation of the feedback gains to obtain adequate control and secondly, that arrangement required much higher control rates, both for pitch and zero commands.

Decoupling for higher mode models.- As the size of the model is increased, the control process becomes more complicated. Results of decoupling, with a control actuator at each end of the beam, are shown for five-mode and six-mode models in figures 14 and 15, respectively. In both cases, the actuators are located exactly at the ends of the beam (special case) and are used to decouple the pitch attitude and the first flexible mode. The B matrices for these cases are composed of the B_2 matrix expanded by an additional one or two rows whose values follow the trend established by the second, third, and fourth rows. Under this special condition, the feedback gain matrices are full order, with the five-mode F composed of a combination of fourth-order and sixth-order dynamics. The six-mode F , which is represented entirely by sixth-order dynamics, takes the form shown in table V. For control actuators at other locations,

the ratio method, previously discussed for the four-mode model, could be applied in adjusting the feedback gains in order to obtain control of all modes.

The pitch-command time histories for the five-mode model in figures 14(a) and 14(b) correspond to the feedback matrix F_6 shown in table I. The difference between the time histories was obtained by increasing the eighth column of F_6 by a factor of 100 which smoothed out the high frequency oscillations. In both cases, control of all modes is achieved, but a relatively long period is required to damp out the initial effects. Improved pitch-response characteristics are obtained (shown in figs. 14(c) and 14(d)) with a feedback gain matrix F_7 calculated for a different closed-loop value of ω_0 . However, the initial effects on the flexible-mode responses are substantially larger.

The time histories in figure 15 show the results for the six-mode model with two control actuators, one located at each end of the beam. The corresponding feedback gains are shown as F_8 in table I. Because of the large time scale, some detail is lost in the time histories for the control forces; therefore, the curve for f_2 is plotted on an expanded time scale in figure 15(b). The plots for f_1 and f_2 are similar.

As shown in figure 15, adequate control is achieved for both a pitch and zero command, with only small initial effects in A_2 and A_4 . However, a relatively long period is required to damp out the system. The high frequency oscillations in A_2 and A_4 have been damped out in figures 15(c) and 15(d) by increasing the ninth column of the feedback gain matrix F_8 by a factor of 100.

Summary of effects of increasing number of modes.— Pitch-command control forces are smaller for the higher mode models (fig. 10). However, the zero-command control forces for the six-mode model (fig. 15) are more than twice as large as those for the four-mode model (fig. 8(b)). The zero-command control rates are about three times as high for the six-mode model. These results suggest that higher order structural models may present more severe control requirements. These requirements may be reduced by changing the closed-loop dynamics, but only to the point where the response characteristics become undesirable.

Change in number of control actuators.— Figure 16 shows decoupling of the six-mode model achieved by three actuators, where the pitch plus the first two flexible modes are decoupled. The control-actuator locations are shown in figure 16(a). As this figure illustrates, a pitch command produces oscillations in the three modes not included in the decoupling control law. (The corresponding feedback gain matrix is given in table I as F_9 .) These oscillations were damped out (figs. 16(b) and 16(c)) by adjusting the feedback gains using the ratio method. The closed-loop value of $\omega_2 = 0.1$ rad/sec was used to produce somewhat better response characteristics for the system.

Use of the ratio method necessitated change of the actual control-influence matrix B_6 (table II) to B_7 in order to obtain the conditions for the special case previously discussed. The flexible-mode coefficients in column 2 were averaged to give the value 0.0005. The B_7 matrix was then used to derive the feedback gain matrix F_{10} (table I) required to adjust the feedback gains in the actual case. F_{11} is the actual feedback gain matrix (adjusted by the ratio

method) which corresponds to the actual B_6 control-influence matrix. The ratio method required that the zero gains originally occurring in the six columns of F_{11} (similar to those shown for F_9) be adjusted in the manner previously described for the four-mode model. For example, the gains in the fourth column were adjusted with respect to those in the second column according to the ratios of corresponding columns in F_{10} , and so on. A change in sign in the eleventh column was also required.

Summary of effect of number of control actuators.- Comparison of figures 15 and 16 indicates two important aspects concerning the selection of the number of control actuators to be employed in decoupling. First, increasing the number of actuators increases the complexity of adjusting the feedback gains for adequate control. Second, the control forces required for a pitch command increase by one to two orders of magnitude for the three-actuator case. (The zero-command forces are essentially unaffected.) The number of actuators should therefore be kept to a minimum.

CONCLUDING REMARKS

A study has been conducted of the feasibility of employing decoupling procedures to control large flexible structures in low Earth orbit. The analysis was performed in connection with a 100-m free-free beam. Control involved commanding changes in pitch attitude without affecting the flexible modes, and nulling initial disturbances in the pitch and flexible modes. Although the analysis was limited to a simple structure, the results obtained could be considered generally applicable to more complex large space structures.

The study investigated two decoupling control methods. For the first method, the system was completely decoupled (that is, the number of control actuators equaled the number of modes in the model). Certain actuators were then eliminated, and the feedback gains were adjusted to maintain control of the system. For the second method, specified modes of the system were excluded from the decoupling control law by employing fewer control actuators than modes in the model. Adjustments were then made in the feedback gains to include the uncontrolled modes in the overall control of the system. Observations based on the study indicate the following:

1. Both methods were capable of controlling the beam; however, the second method was superior because it employed a simplified gain adjustment procedure.
2. Both methods produced initial effects on the system response characteristics; however, the effects were generally small and were damped out in relatively short time periods.
3. Control systems designed by incomplete decoupling were found to be generally insensitive to uncertainties in the model parameters. This characteristic can be used to manipulate control models to obtain proper feedback gains for overall control of the system.

4. Control-force requirements depend on many factors associated with the design of the system. As expected, the location of the control actuators has a large effect. The requirements generally increase with an increase in the number of actuators and decrease as the number of modes included in the model is increased.

By far, the most important factor influencing control-force requirements is the selection of the closed-loop value of pitch angular frequency for the system. The requirements increase rapidly with increases in this value.

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September 30, 1980

APPENDIX

EXAMPLE OF DECOUPLED CONTROL

An example of decoupled control for the pitch and first flexible modes is presented for a four-mode model (pitch plus first three flexible modes). Two control actuators are required, one located at each end of the beam, and, referring to equation (8):

$$W = \begin{bmatrix} -3 \times 10^{-6} & 0 & 0 & 0 \\ 0 & -0.0032 & 0 & 0 \\ 0 & 0 & -0.0288 & 0 \\ 0 & 0 & 0 & -0.09308 \end{bmatrix} \quad (A1)$$

$$B = \begin{bmatrix} 0.00005952 & -0.0000590 \\ 0.0020 & 0.00201 \\ 0.0020 & -0.0020 \\ 0.0020 & 0.0020 \end{bmatrix} \quad (A2)$$

The decoupled control law is taken as

$$\bar{f} = F\bar{x} + G\bar{v} \quad (A3)$$

where \bar{v} is the input command vector, and F and G are the feedback and feedforward gain matrices, respectively.

For decoupled control of the pitch mode and the first flexible mode, the output equation is

$$\bar{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \bar{x} \quad (A4)$$

where \bar{y} is the output vector (pitch angle and first modal amplitude).

APPENDIX

The decoupling control law gains F and G are determined as follows:
For pitch control only, using equations (A1), (A2), and (A4),

$$y_1 = \theta \quad \dot{y}_1 = \dot{\theta} \quad \ddot{y}_1 = \ddot{\theta}$$

$$\ddot{y}_1 = -0.000003\theta + 0.00005952f_1 - 0.0000590f_2 \quad (A5)$$

To obtain desired second-order response dynamics, augmented frequency and damping terms are added to both sides of equation (A5), so that

$$\ddot{y}_1 + 2\zeta_\theta\omega_\theta\dot{y}_1 + \omega_\theta^2 y_1 = (\omega_\theta^2 - 0.000003)\theta + 2\zeta_\theta\omega_\theta\dot{\theta}$$

$$+ 0.00005952f_1 - 0.0000590f_2 = v_1 \quad (A6)$$

Values for the quantities ω_θ and ζ_θ can be selected to provide feedback gains for a desired dynamic response in pitch. Similarly, for first flexible mode control only

$$y_2 = A_1 \quad \dot{y}_2 = \dot{A}_1 \quad \ddot{y}_2 = \ddot{A}_1$$

$$\ddot{y}_2 = -0.0032A_1 + 0.0020f_1 + 0.00201f_2 \quad (A7)$$

To obtain desired second-order response dynamics, adding the augmented terms in equation (A7) gives

$$\ddot{y}_2 + 2\zeta_1\omega_1\dot{y}_2 + \omega_1^2 y_2 = (\omega_1^2 - 0.0032)A_1 + 2\zeta_1\omega_1\dot{A}_1$$

$$+ 0.0020f_1 + 0.00201f_2 = v_2 \quad (A8)$$

where values for ω_1 and ζ_1 can be selected to provide feedback gains for a desired dynamic response in the first flexible mode.

The equations (A6) and (A8) written in matrix form are

$$\bar{v} = M\bar{x} + N\bar{f}$$

or

$$\bar{f} = -N^{-1}M\bar{x} + N^{-1}\bar{v}$$

APPENDIX

Provided N is not singular, then

$$\bar{f} \triangleq F\bar{x} + G\bar{v}$$

which is the required decoupled control law (eq. (A3)), where

$$F = -N^{-1}M = F' + F''$$

and

$$G = N^{-1}$$

as the required feedback and feedforward gain matrices, respectively. The feedforward gains operate on the command inputs, and the feedback gains operate on the amplitude and rate measurements of all the states. The feedback gain matrix is separated into two parts: that part F' which deletes the unaugmented dynamics, and F'' which incorporates the dynamics selected for the closed-loop system.

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TABLE I.- FEEDBACK GAIN MATRICES USED IN ANALYSIS

$$F_1 = \begin{bmatrix} -73.0467 & -0.8554 & 0.6119 & 10.3187 & -730.6858 & -12.5798 & -3.2548 & -12.4202 \\ -154.8164 & 0 & -8.6644 & 0 & -1548.6289 & 0 & 46.0872 & 0 \\ -57.4491 & 2.4131 & -3.2152 & 29.4819 & -574.6633 & 35.4862 & 17.1020 & -35.4862 \\ 17.5055 & -0.8554 & -3.7203 & 10.3187 & 175.1072 & -12.5798 & 19.7888 & -12.4202 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -29.0939 & -159.6918 & -14.1334 & -214.5082 & -1166.5128 & -1372.9417 & -65.2846 & 872.9447 \\ 29.0939 & -159.6918 & 14.1334 & -214.5082 & 1166.5128 & -1372.9417 & 65.2846 & 872.9447 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -84.5581 & -61.2754 & -41.0771 & -82.3089 & -1691.6686 & -248.2797 & -94.6755 & 157.8616 \\ 84.1374 & -61.8155 & 40.8727 & -83.0343 & 1683.2523 & -250.4679 & 94.2045 & 159.2529 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 41.7280 & -154.0575 & -20.2710 & -206.9400 & 834.8100 & -624.2197 & -46.7206 & -396.8920 \\ 419.3766 & -308.1150 & -203.7276 & -413.8694 & 8390.0500 & -1248.4395 & -469.5540 & -793.7840 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} 14.4561 & -398.7301 & -7.0226 & -535.6011 & 579.6156 & -3428.0692 & -32.4385 & -2179.6372 \\ 145.2877 & -797.4621 & -70.5789 & -1071.2022 & 5825.2822 & -6856.1386 & -326.0154 & -4359.2745 \end{bmatrix}$$

$$F_6 = \begin{bmatrix} -1.1122 & -159.6918 & 3.1508 & -214.5082 & -40.6170 & -68.4826 & -1372.9417 & -0.7027 & 872.9417 & -147.2593 \\ 1.1122 & -159.6918 & -3.1508 & -214.5082 & 40.6170 & 68.4826 & -1372.9417 & 0.7027 & 872.9417 & 147.2593 \end{bmatrix}$$

TABLE I.- Concluded

$$F_7 = \begin{bmatrix} -832.4541 & -159.6918 & -63.2516 & -214.5082 & -249.4739 & -16656.0630 & -1372.9417 & 19.9662 & 872.9417 & 25.7182 \\ 832.4541 & -159.6918 & 63.2516 & -214.5082 & 249.4739 & 16656.0630 & -1372.9417 & -19.9662 & 872.9417 & -25.7182 \end{bmatrix}$$

$$F_8 = \begin{bmatrix} -1.1122 & -54.5126 & 3.1508 & -199.0957 & -40.6170 & -683.0917 & -68.4826 & -784.1744 & -0.7027 & -97.6885 & -147.2593 & 131.8629 \\ 1.1122 & -54.5126 & -3.1508 & -199.0957 & 40.6170 & -683.0917 & 68.4826 & -784.1744 & 0.7027 & -97.6885 & 147.2593 & 131.8629 \end{bmatrix}$$

$$F_9 = \begin{bmatrix} -37.5461 & -61.7000 & -30.5763 & 0 & 0 & 0 & -751.1479 & -250.0000 & -138.2292 & 0 & 0 & 0 \\ 154.4491 & 0 & -102.0060 & 0 & 0 & 0 & 3099.1146 & 0 & -461.1483 & 0 & 0 & 0 \\ 52.9596 & -61.7000 & 20.4267 & 0 & 0 & 0 & 1059.5098 & -250.0000 & 92.3449 & 0 & 0 & 0 \end{bmatrix}$$

$$F_{10} = \begin{bmatrix} 0 & -54.5126 & 8.9967 & -199.0957 & -16.7967 & -683.0917 & 0 & -784.1744 & 8.2991 & -97.6884 & -108.2991 & 131.8628 \\ 223.9471 & 0 & 23.9911 & 0 & -44.7911 & 0 & 4480.2870 & 0 & 22.1310 & 0 & -288.7977 & 0 \\ 55.9868 & -54.5126 & -2.9989 & -199.0957 & 5.5989 & -683.0917 & 1120.0720 & -784.1744 & -2.7664 & -97.6884 & 36.0997 & 131.8628 \end{bmatrix}$$

$$F_{11} = \begin{bmatrix} -37.5461 & -61.7000 & 2.5987 & -225.3460 & -4.8518 & -773.1558 & -751.1479 & -250.0000 & -27.6458 & -31.1437 & -360.7636 & 42.0387 \\ 154.4491 & 0 & 8.6696 & 0 & -16.1860 & 0 & 3099.1146 & 0 & -92.2297 & 0 & -1203.5481 & 0 \\ 52.9596 & -61.7000 & -1.7361 & -225.3460 & 3.2412 & -773.1558 & 1059.5098 & -250.0000 & 18.4690 & -31.1437 & 241.0108 & 42.0387 \end{bmatrix}$$

TABLE II.- CONTROL-INFLUENCE MATRICES USED IN ANALYSIS

$$B_1 = \begin{bmatrix} 0.00005952 & 0.00002976 & 0 & -0.00005952 \\ 0.002 & -0.000198 & -0.0014 & 0.002 \\ 0.002 & -0.0011698 & 0 & -0.002 \\ 0.002 & -0.0012437 & 0.001418 & 0.002 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.00005952 & -0.00005952 \\ 0.002 & 0.002 \\ 0.002 & -0.002 \\ 0.002 & 0.002 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0.00005952 & -0.0000590 \\ 0.002 & 0.00201 \\ 0.002 & -0.002 \\ 0.002 & 0.002 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0.00006547 & -0.00007142 \\ 0.0018 & 0.0016 \\ 0.0022 & -0.0016 \\ 0.0024 & 0.0018 \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 0.00005952 & -0.00002976 \\ 0.002 & -0.000199 \\ 0.002 & 0.0011685 \\ 0.002 & -0.001237 \end{bmatrix}$$

$$B_6 = \begin{bmatrix} 0.00005952 & -0.00002976 & -0.00005952 \\ 0.002 & -0.000199 & 0.002 \\ 0.002 & 0.0011685 & -0.002 \\ 0.002 & -0.001237 & 0.002 \\ 0.002 & 0.000501 & -0.002 \\ 0.002 & -0.00052 & 0.002 \end{bmatrix}$$

$$B_7 = \begin{bmatrix} 0.00005952 & -0.00002976 & -0.00005952 \\ 0.002 & -0.0005 & 0.002 \\ 0.002 & 0.0005 & -0.002 \\ 0.002 & -0.0005 & 0.002 \\ 0.002 & 0.0005 & -0.002 \\ 0.002 & -0.0005 & 0.002 \end{bmatrix}$$

TABLE III.- F REQUIRED TO CHANGE DYNAMICS FOR FOUR-MODE MODEL

[Two control actuators; subscript 1 denotes first flexible mode]

$$a_{F''}^{\text{exact}} = \begin{bmatrix} 8400 & 250 \\ -8400 & 250 \end{bmatrix} \begin{bmatrix} \frac{s_{\theta} - Q_{\theta}\omega_{\theta u}}{\omega_{2u} - \omega_{\theta u}} & 0 & 0 & 0 \\ 0 & \frac{s_1 - Q_1\omega_{1u}}{\omega_{3u} - \omega_{1u}} & 0 & 0 \end{bmatrix} + \begin{bmatrix} 250 & 250 \\ -250 & 250 \end{bmatrix} \begin{bmatrix} 0 & 0 & \frac{-s_{\theta} + Q_{\theta}\omega_{2u}}{\omega_{2u} - \omega_{\theta u}} & 0 \\ 0 & 0 & 0 & \frac{-s_1 + Q_1\omega_{3u}}{\omega_{3u} - \omega_{1u}} \end{bmatrix}$$

$$(s^2 + 2\zeta\omega s + \omega^2)^2 = s^4 + 4\zeta\omega s^3 + 2\omega^2(1 + 2\zeta^2)s^2 + 4\zeta\omega^3s + \omega^4$$

$$= s^4 + Ps^3 + Qs^2 + Rs + S$$

$$F''_{\text{offset}} = \begin{bmatrix} 8458 & 248.3 \\ -8416 & 250.5 \end{bmatrix} \begin{bmatrix} K_{\theta} & 0 & 0 & 0 & D_{\theta} & 0 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & D_1 & 0 & 0 \end{bmatrix}$$

$$s^2 + 2\zeta\omega s + \omega^2 = s^2 + Ds + K$$

^aFirst four columns only shown.

TABLE IV.- COMPARISON OF CONTROL-FORCE REQUIREMENTS FOR
PITCH AND ZERO COMMANDS

[Four-mode model with one control actuator at each end of beam]

ω_θ	ζ_θ	0.01-rad pitch command	Zero command, $\theta = A_1 = A_2 = A_3 = 0.01$			
			$\omega_1 = 0.1 \text{ rad/sec}, \zeta_1 = 1.0$		$\omega_1 = 0.5 \text{ rad/sec}, \zeta_1 = 1.0$	
		$f_1, -f_2$	f_1	f_2	f_1	f_2
0.1	0.1	0.29	-0.48	$-(f_1 + 0.28)$	-4.08	$-(f_1 + 7.48)$
.1	.5	.29	-.50	↓	-4.10	↓
.1	1.0	.29	-.57	↓	-4.17	↓
.5	1.0	182.2	-180.8	↓	-184.4	↓
1.0	1.0	2917	-2845	↓	-2849	↓

TABLE V.- F REQUIRED TO CHANGE DYNAMICS FOR SIX-MODE MODEL

[Two control actuators; subscript 1 denotes first flexible mode]

$$a_{F''}''_{\text{exact}} = \begin{bmatrix} 8400 & 250 \\ -8400 & 250 \end{bmatrix} \begin{bmatrix} \frac{U_{\theta} - S_{\theta}\omega_{\theta u} + Q_{\theta}\omega_{\theta u}^2}{(\omega_{2u} - \omega_{\theta u})(\omega_{4u} - \omega_{\theta u})} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{U_1 - S_1\omega_{1u} + Q_1\omega_{1u}^2}{(\omega_{3u} - \omega_{1u})(\omega_{5u} - \omega_{1u})} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 250 & 250 \\ -250 & 250 \end{bmatrix} \begin{bmatrix} 0 & 0 & \frac{-U_{\theta} + S_{\theta}\omega_{2u} - Q_{\theta}\omega_{2u}^2}{(\omega_{2u} - \omega_{\theta u})(\omega_{4u} - \omega_{2u})} & 0 & \frac{U_{\theta} - S_{\theta}\omega_{4u} + Q_{\theta}\omega_{4u}^2}{(\omega_{4u} - \omega_{\theta u})(\omega_{4u} - \omega_{2u})} & 0 \\ 0 & 0 & 0 & \frac{-U_1 + S_1\omega_{3u} - Q_1\omega_{3u}^2}{(\omega_{3u} - \omega_{1u})(\omega_{5u} - \omega_{3u})} & 0 & \frac{U_1 - S_1\omega_{5u} + Q_1\omega_{5u}^2}{(\omega_{5u} - \omega_{1u})(\omega_{5u} - \omega_{3u})} \end{bmatrix}$$

$$(s^2 + 2\zeta\omega s + \omega^2)^3 = s^6 + 6\zeta\omega s^5 + 3\omega^2(1 + 4\zeta^2)s^4 + 8\zeta\omega^3(1.5 + \zeta^2)s^3 + 3\omega^4(1 + 4\zeta^2)s^2 + 6\zeta\omega^5s + \omega^6$$

$$= s^6 + Ps^5 + Qs^4 + Rs^3 + Ss^2 + Ts + U$$

^aFirst six columns only shown.

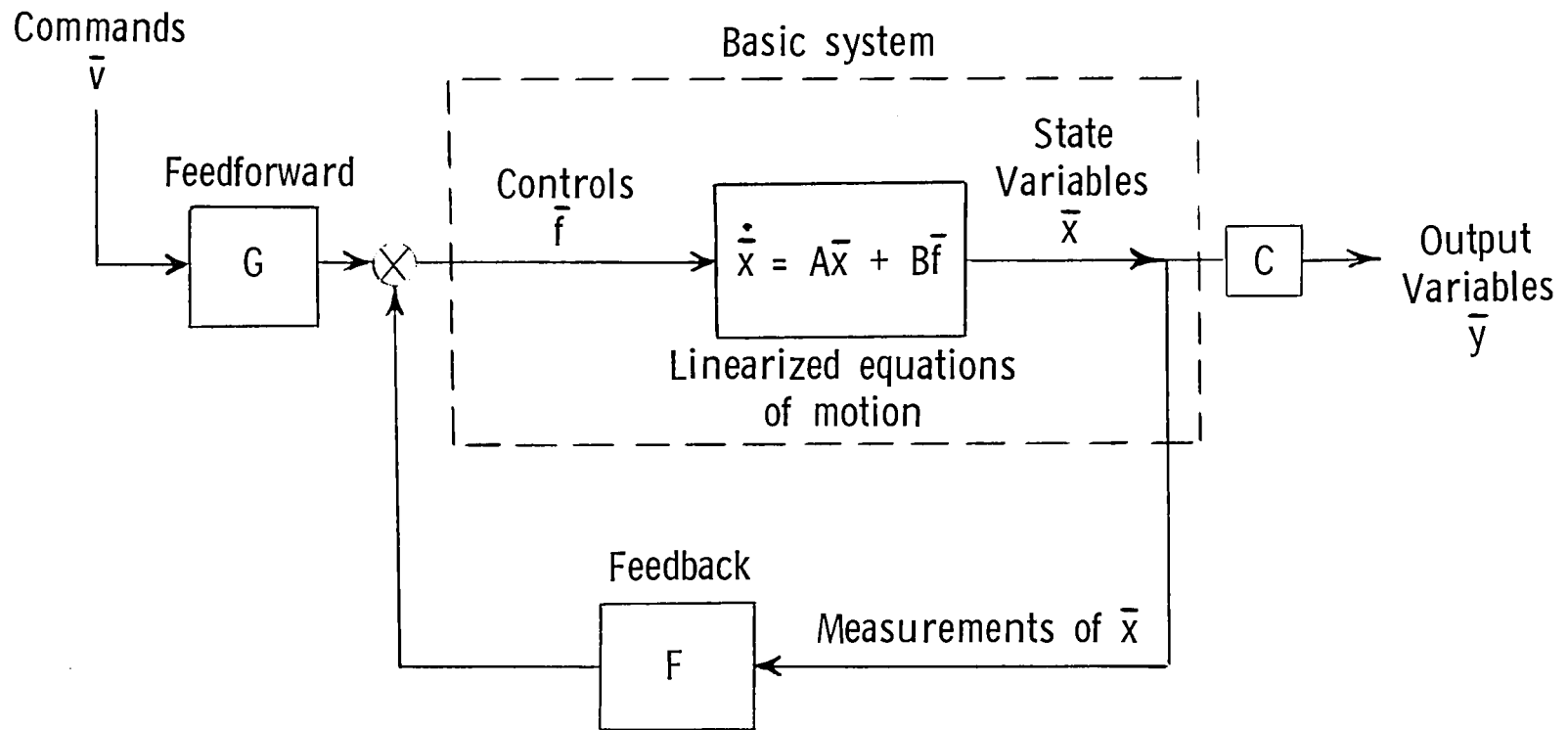
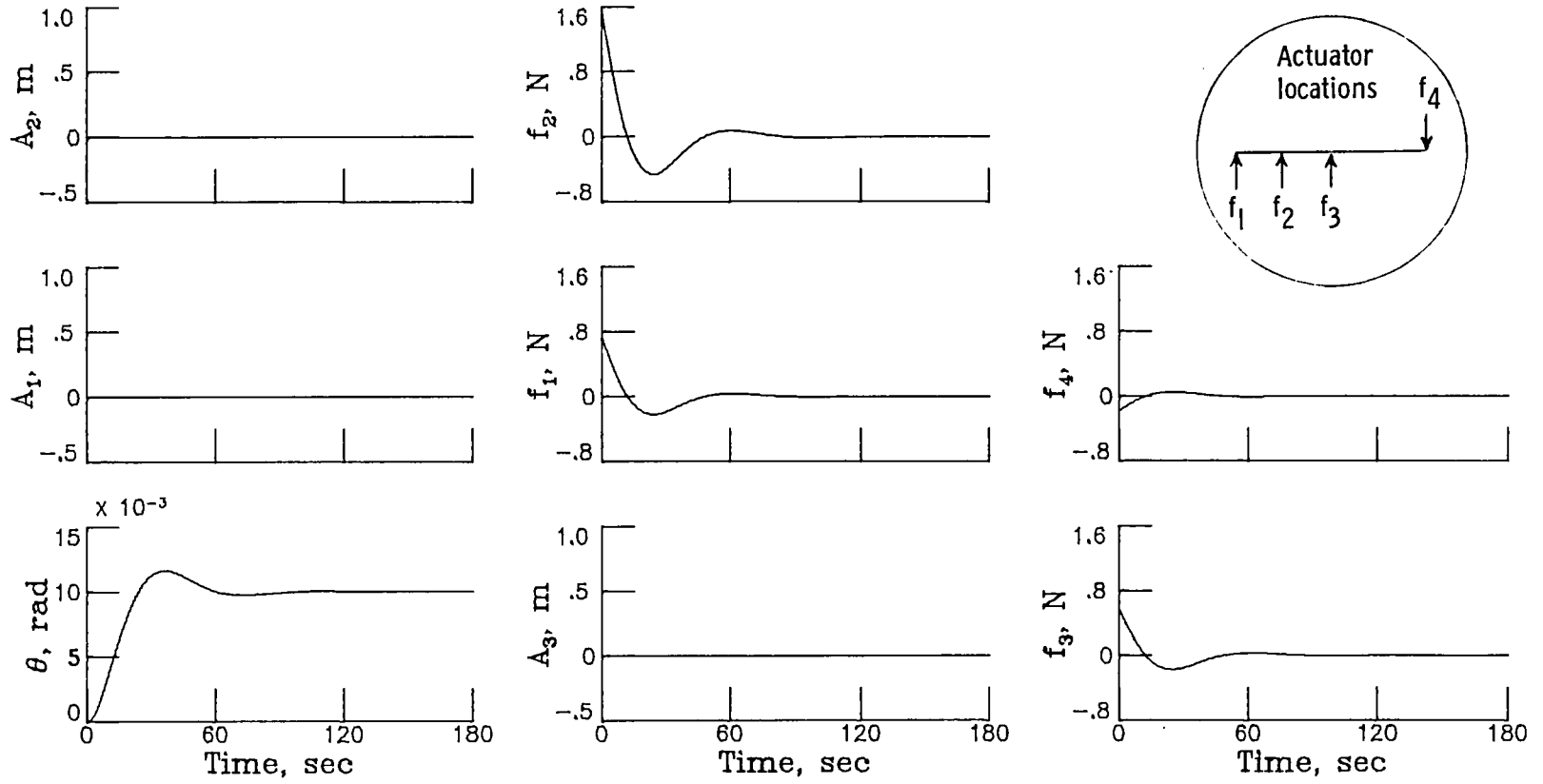


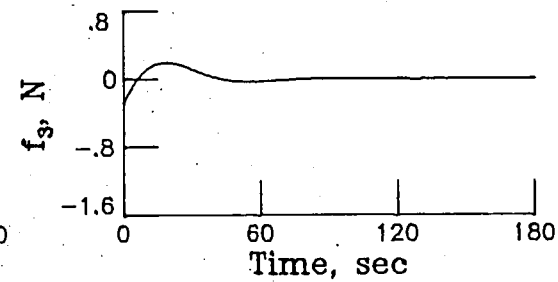
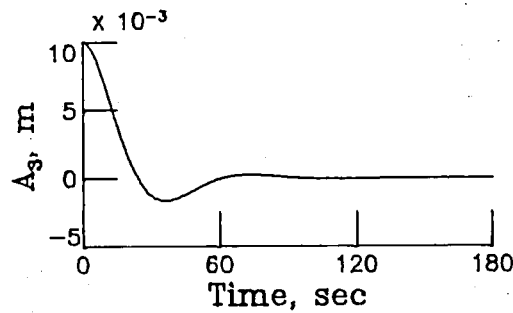
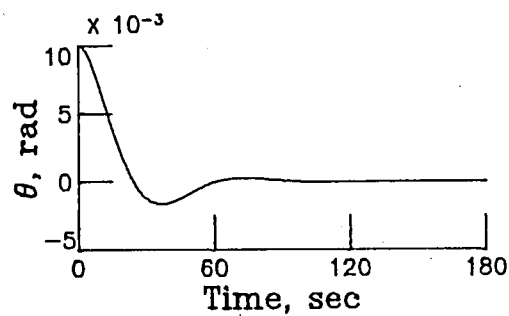
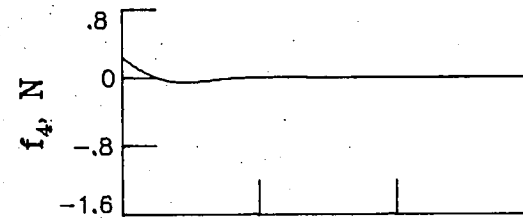
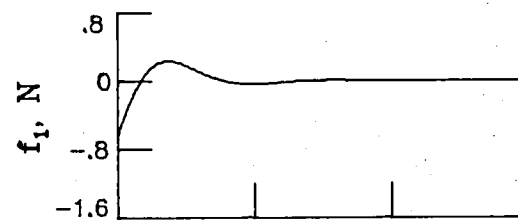
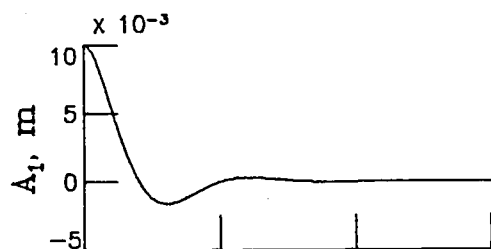
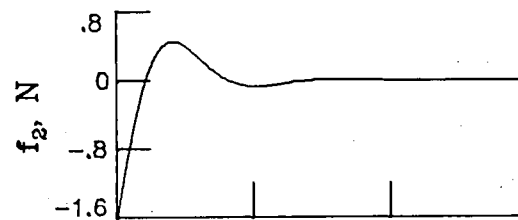
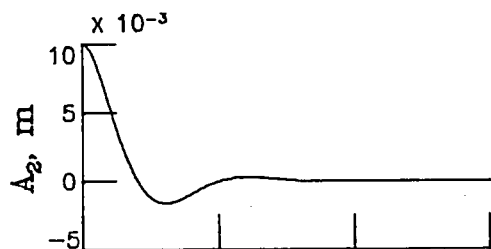
Figure 1.- Diagram of decoupled control system.



(a) Pitch command.

Figure 2.- Response characteristics for completely decoupled control model.

$$\omega_0, \omega_1, \omega_2, \omega_3 = 0.1 \text{ rad/sec}; \quad \zeta_0, \zeta_1, \zeta_2, \zeta_3 = 0.5.$$



(b) Zero command.

Figure 2.- Concluded.

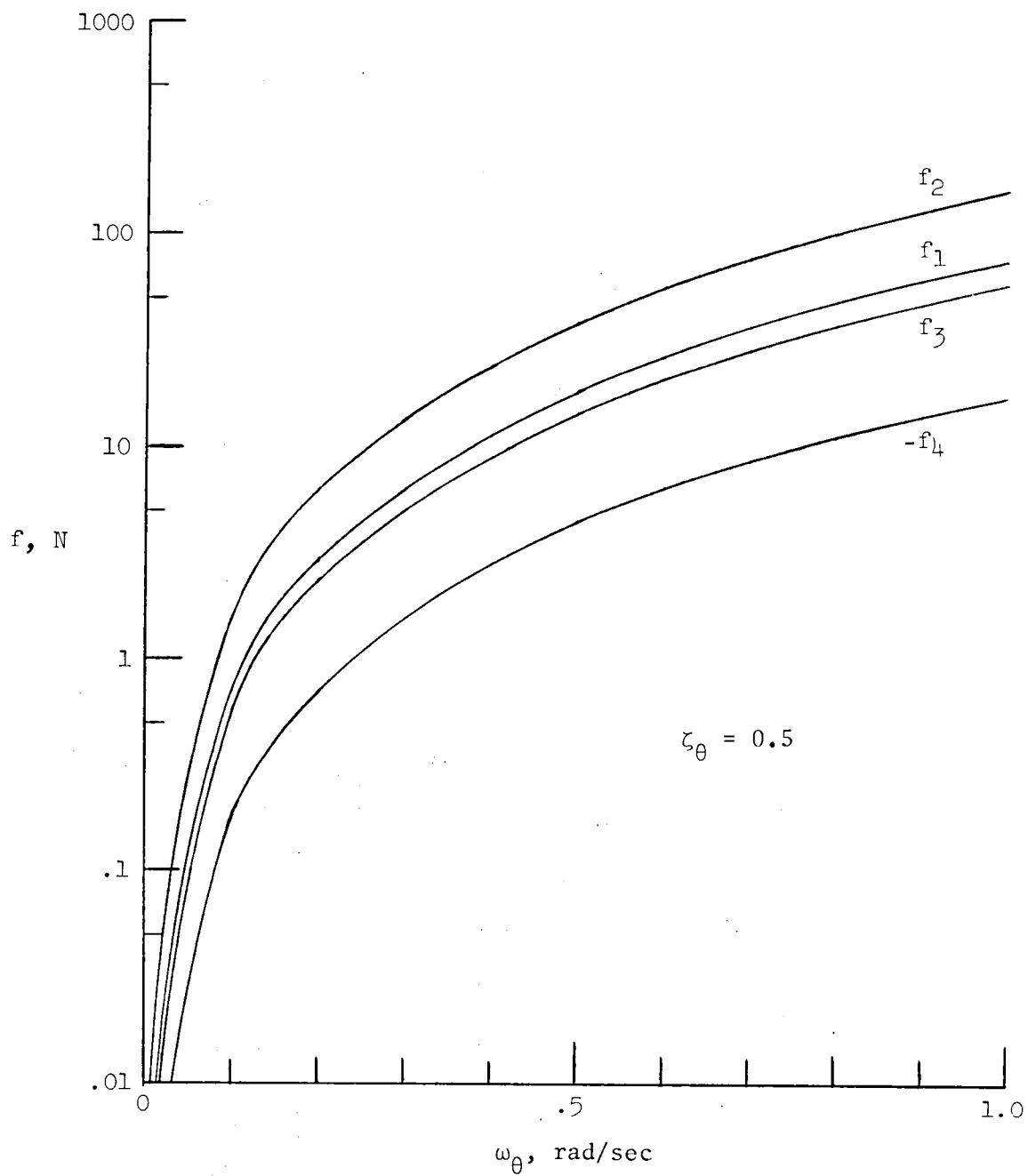
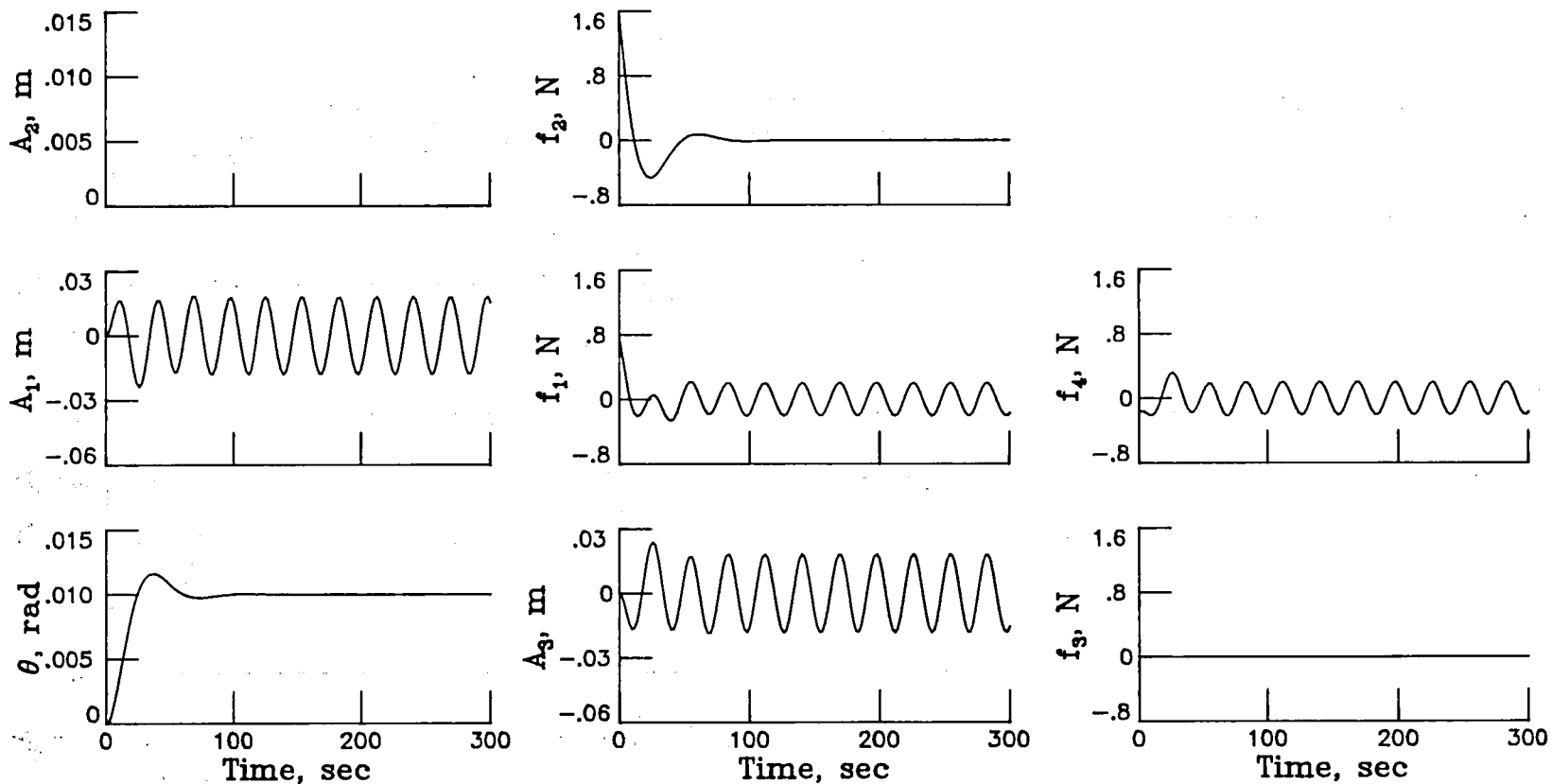
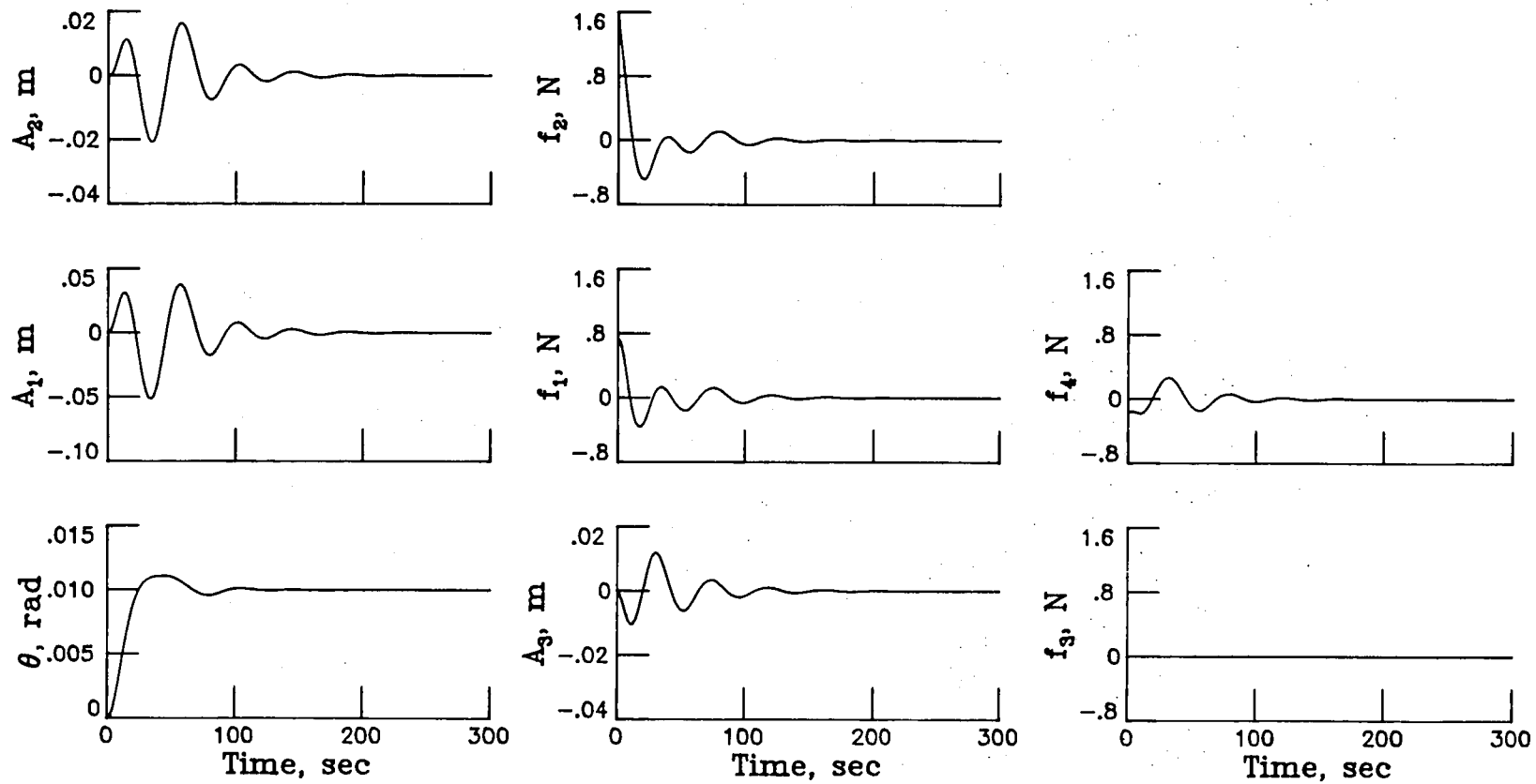


Figure 3.- Control-force requirements for 0.01-rad pitch command. Completely decoupled model.



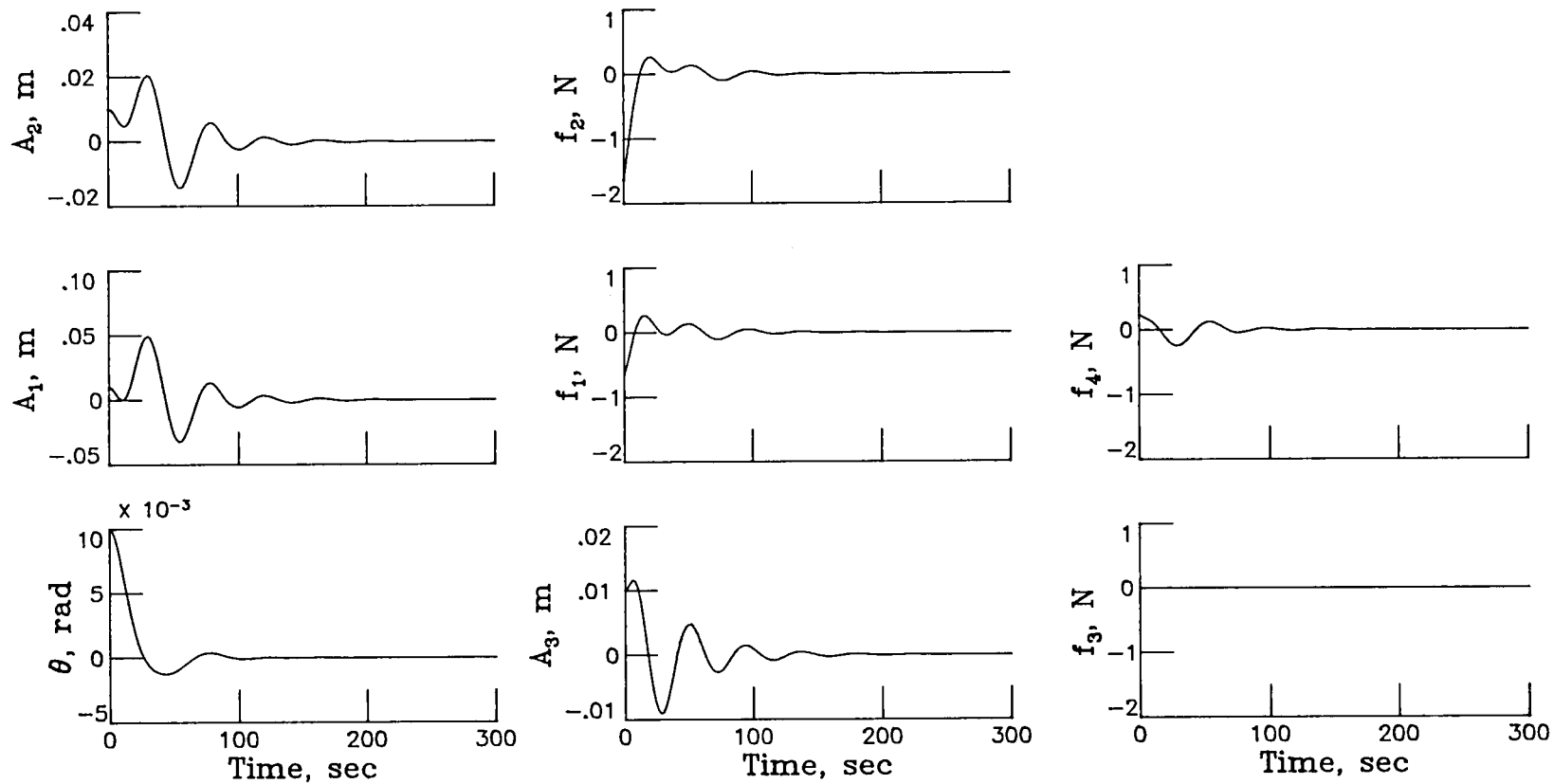
(a) Pitch command; feedback not adjusted.

Figure 4.- Response characteristics for completely decoupled control model with one control actuator eliminated. $\omega_0, \omega_1, \omega_2, \omega_3 = 0.1$ rad/sec; $\zeta_0, \zeta_1, \zeta_2, \zeta_3 = 0.5$.



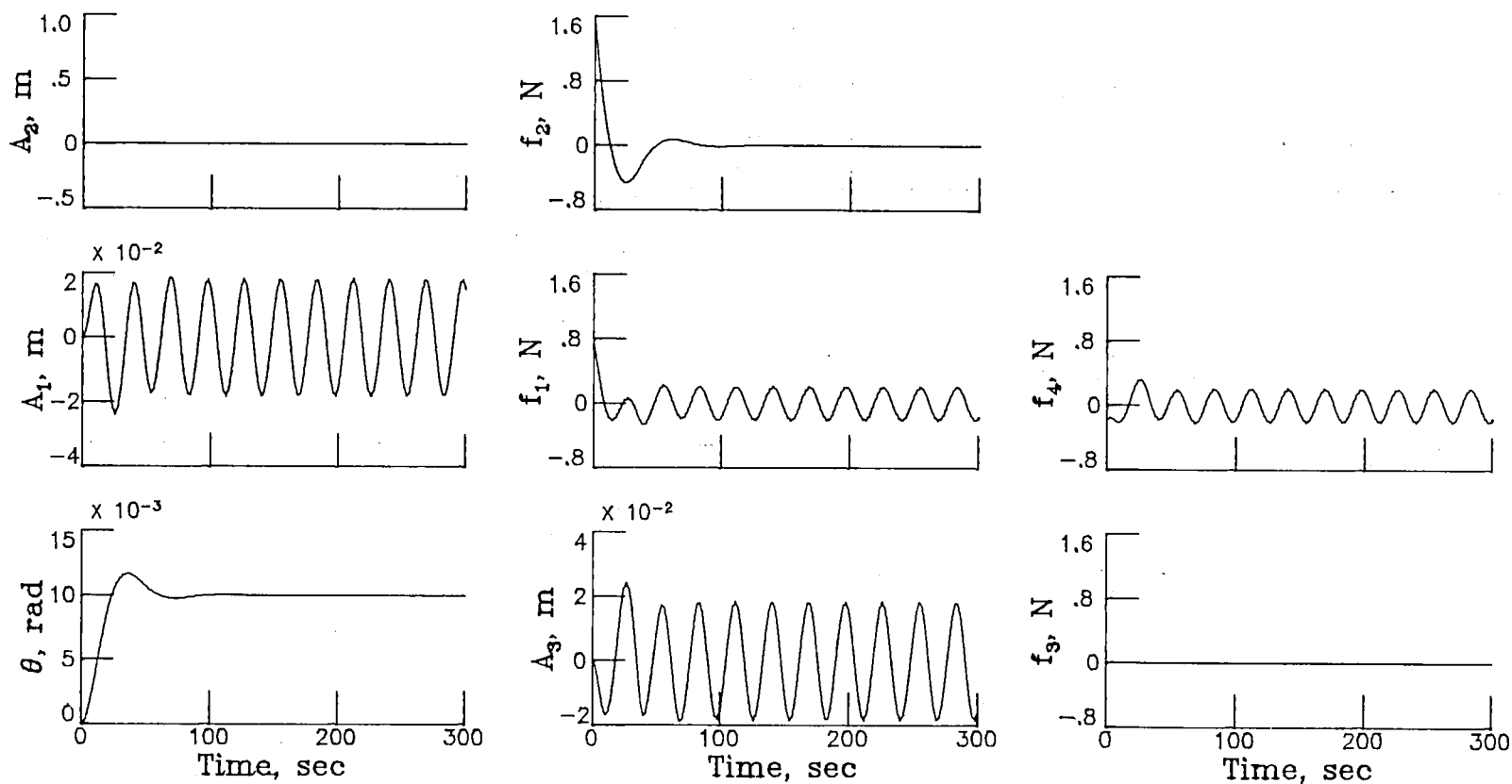
(b) Pitch command; feedback adjusted.

Figure 4.- Continued.



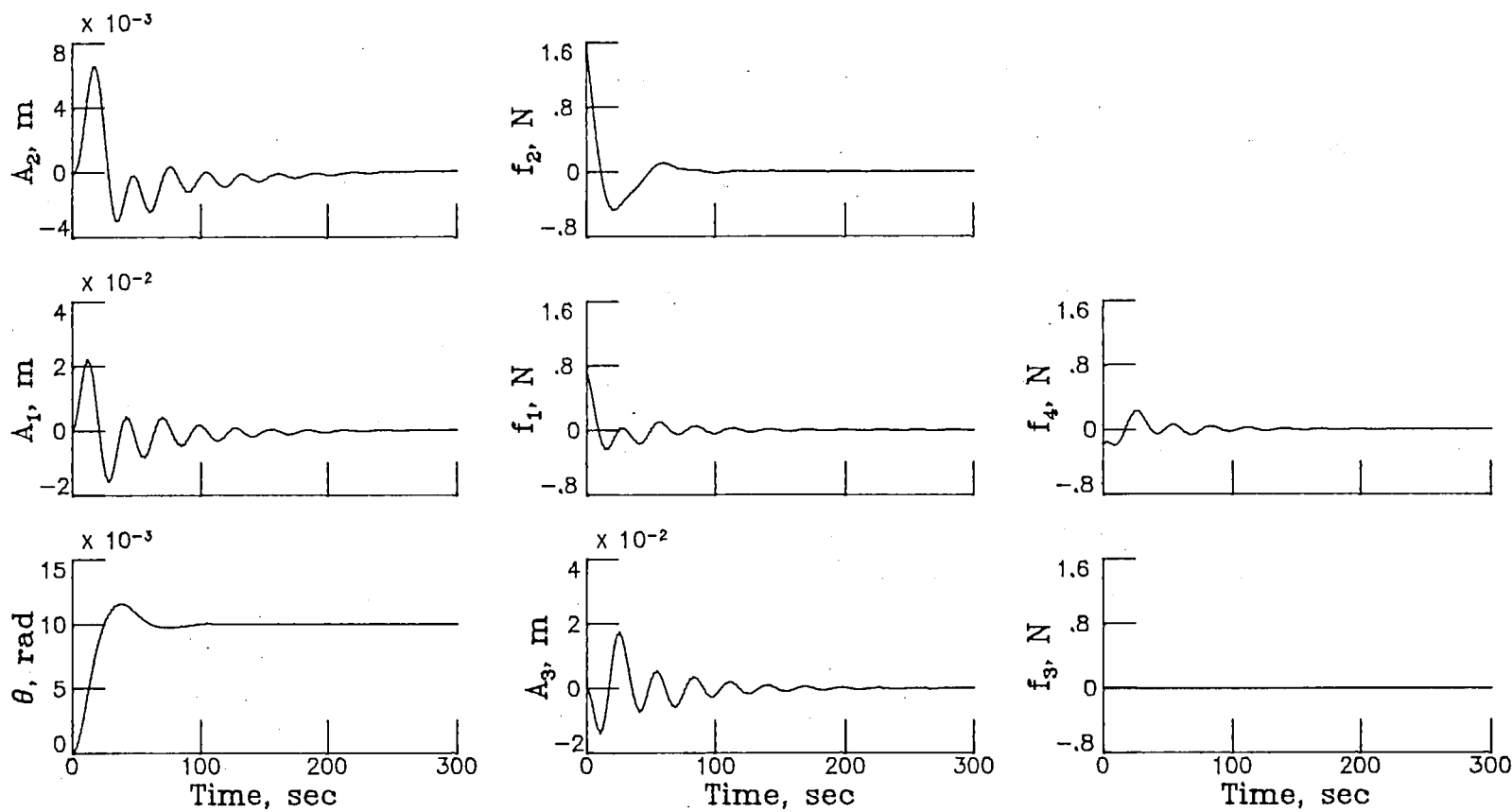
(c) Zero command; feedback adjusted.

Figure 4.- Concluded.



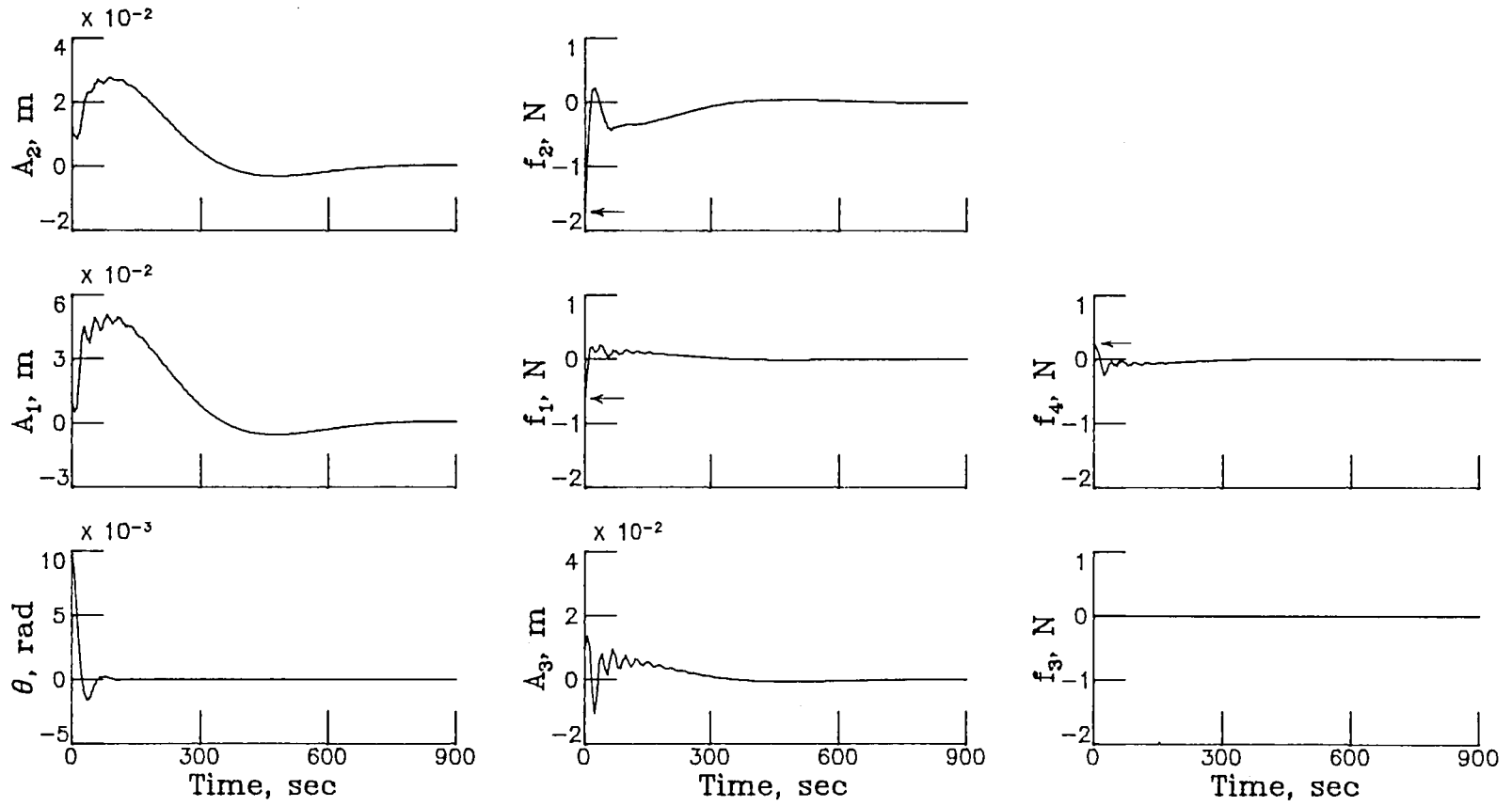
(a) Pitch command; feedback not adjusted.

Figure 5.- Response characteristics for completely decoupled control model with one control actuator eliminated. $\omega_\theta = 0.1$ rad/sec; $\omega_1, \omega_2, \omega_3 = 0.01$ rad/sec; $\zeta_\theta = 0.5$; $\zeta_1, \zeta_2, \zeta_3 = 0.05$. Arrows indicate initial values.



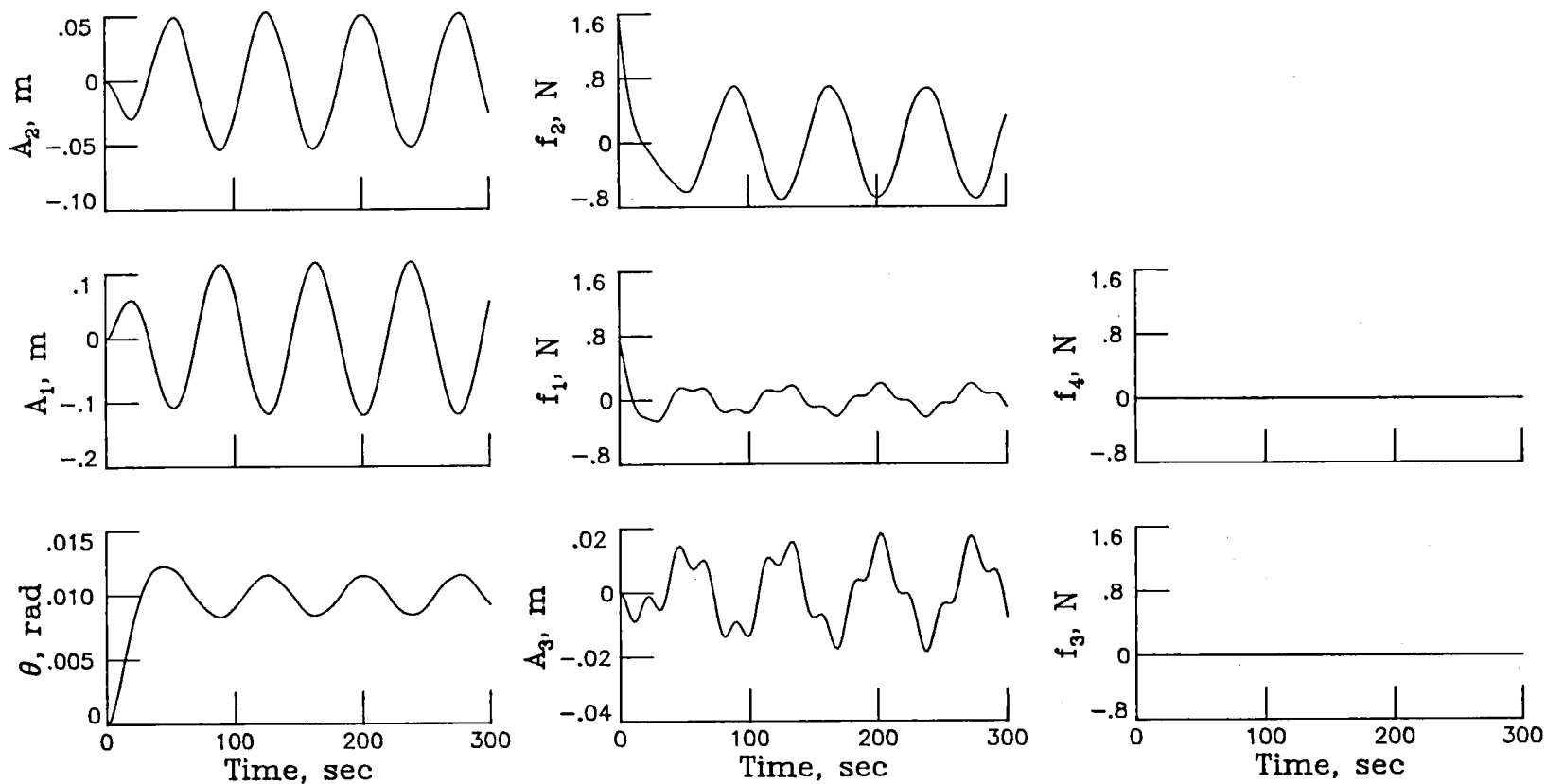
(b) Pitch command; feedback adjusted.

Figure 5.- Continued.



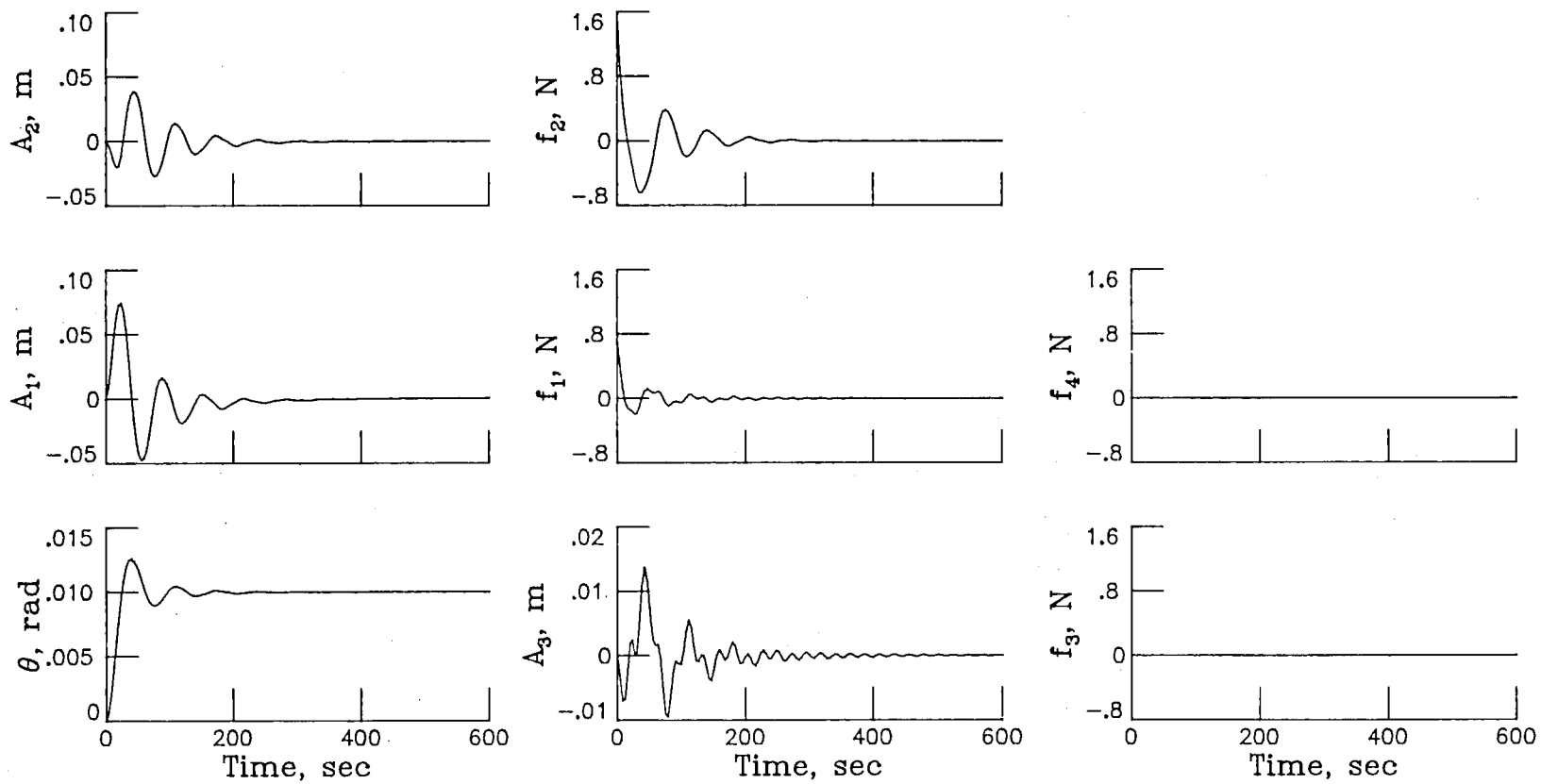
(c) Zero command; feedback adjusted.

Figure 5.- Concluded.



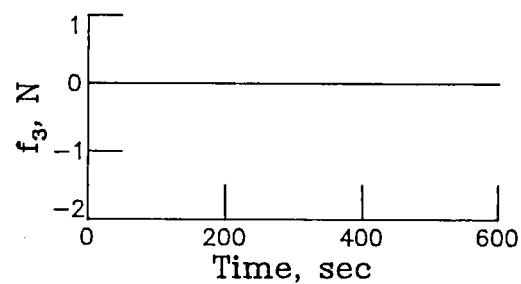
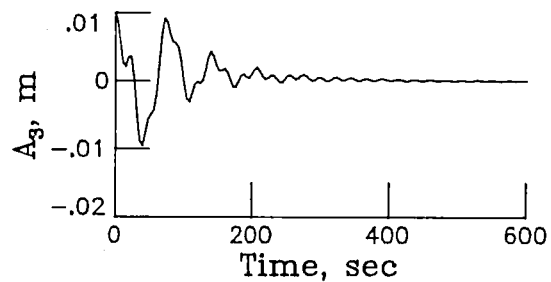
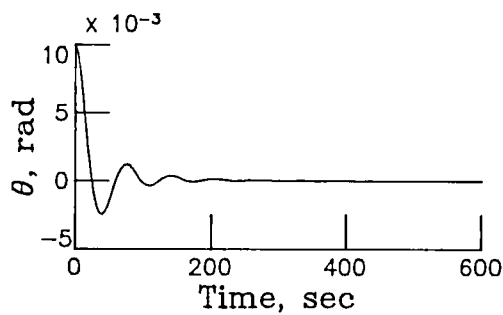
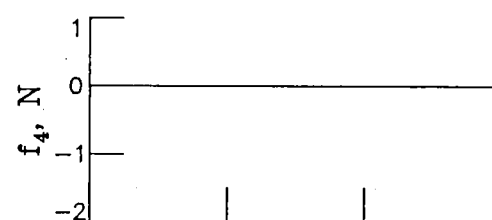
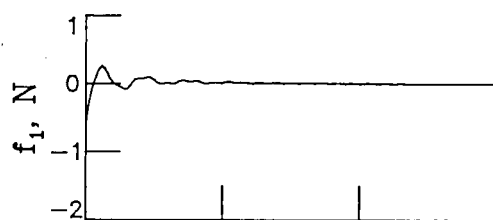
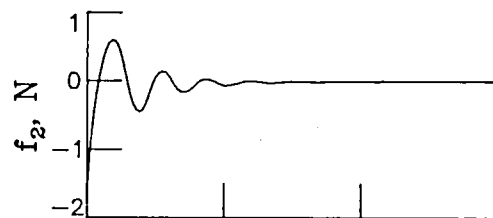
(a) Pitch command; feedback not adjusted.

Figure 6.- Response characteristics for completely decoupled control model with two control actuators eliminated. $\omega_\theta, \omega_1, \omega_2, \omega_3 = 0.1$ rad/sec; $\zeta_\theta, \zeta_1, \zeta_2, \zeta_3 = 0.5$.



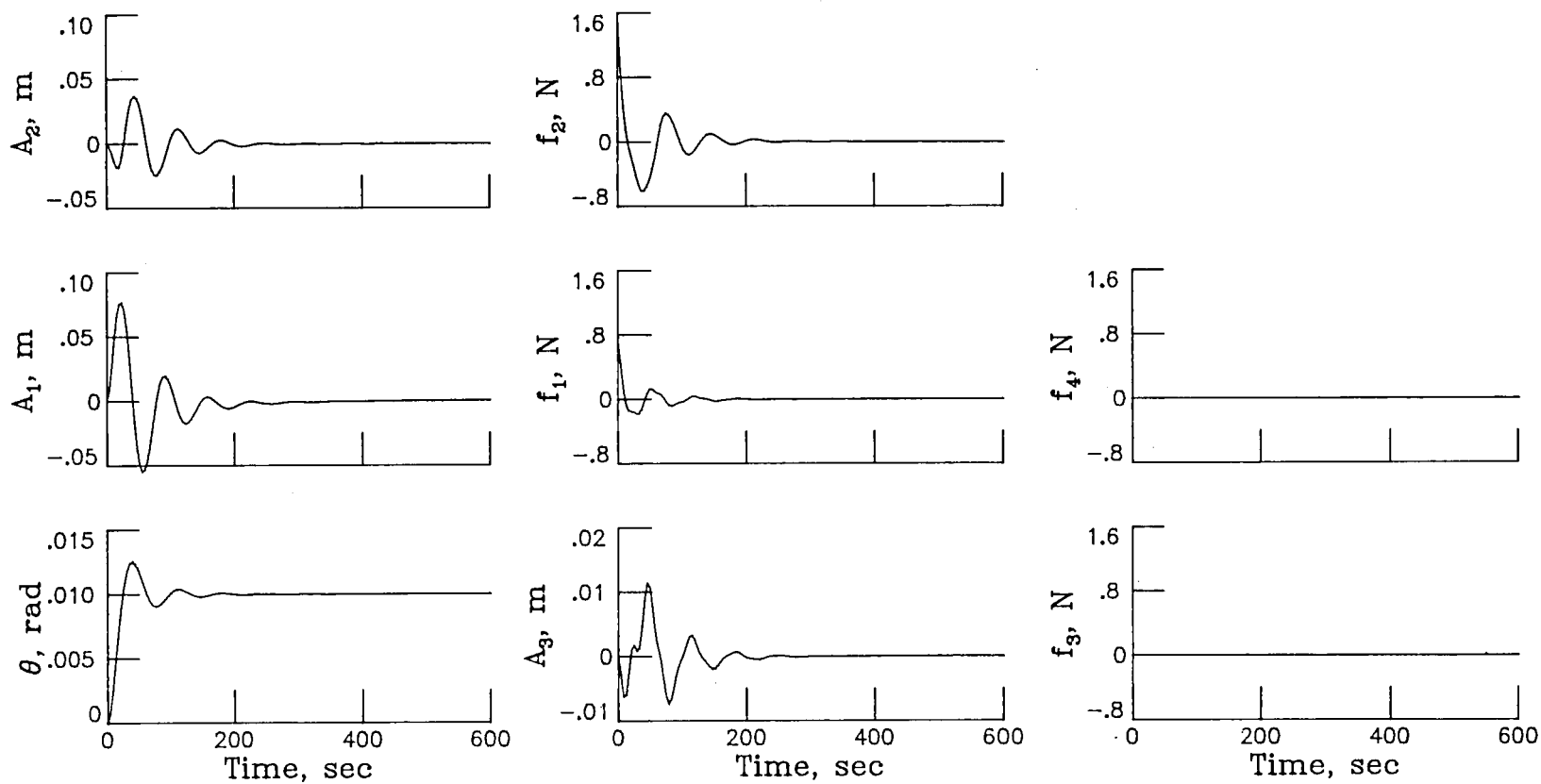
(b) Pitch command; feedback adjusted.

Figure 6.- Continued.



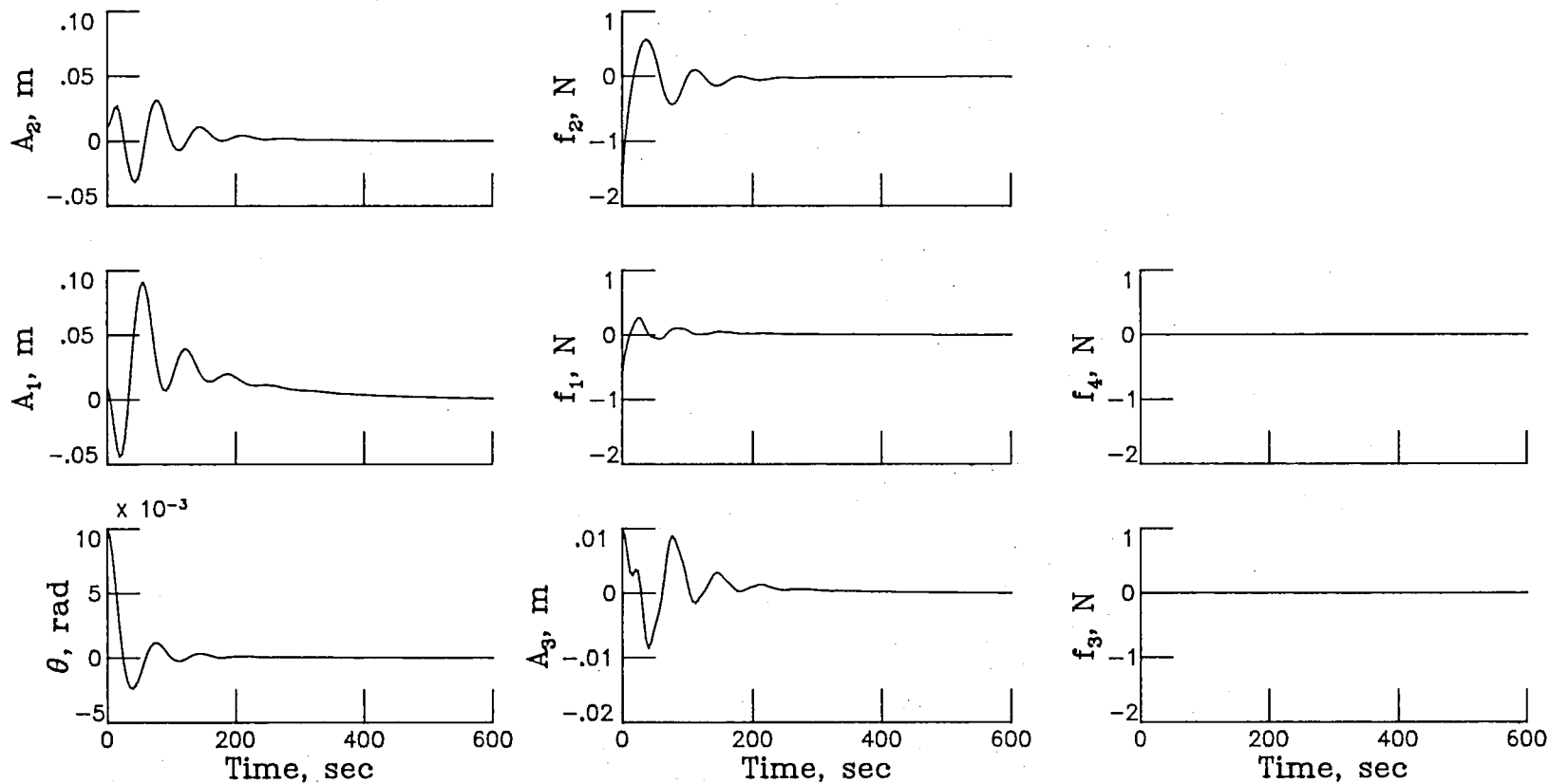
(c) Zero command; feedback adjusted.

Figure 6.- Continued.



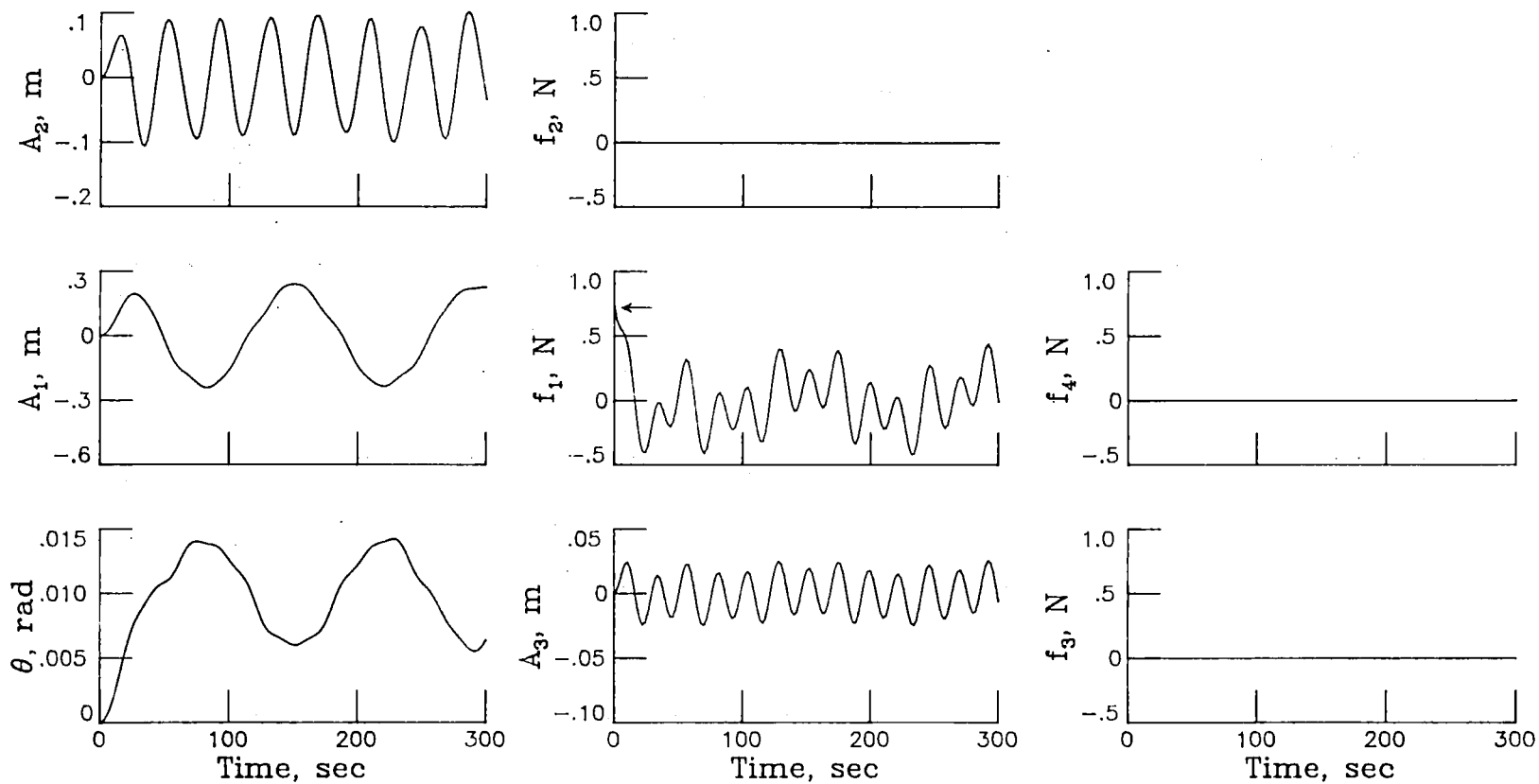
(d) Pitch command; $F_{1,8}$ gain value readjusted.

Figure 6.- Continued.



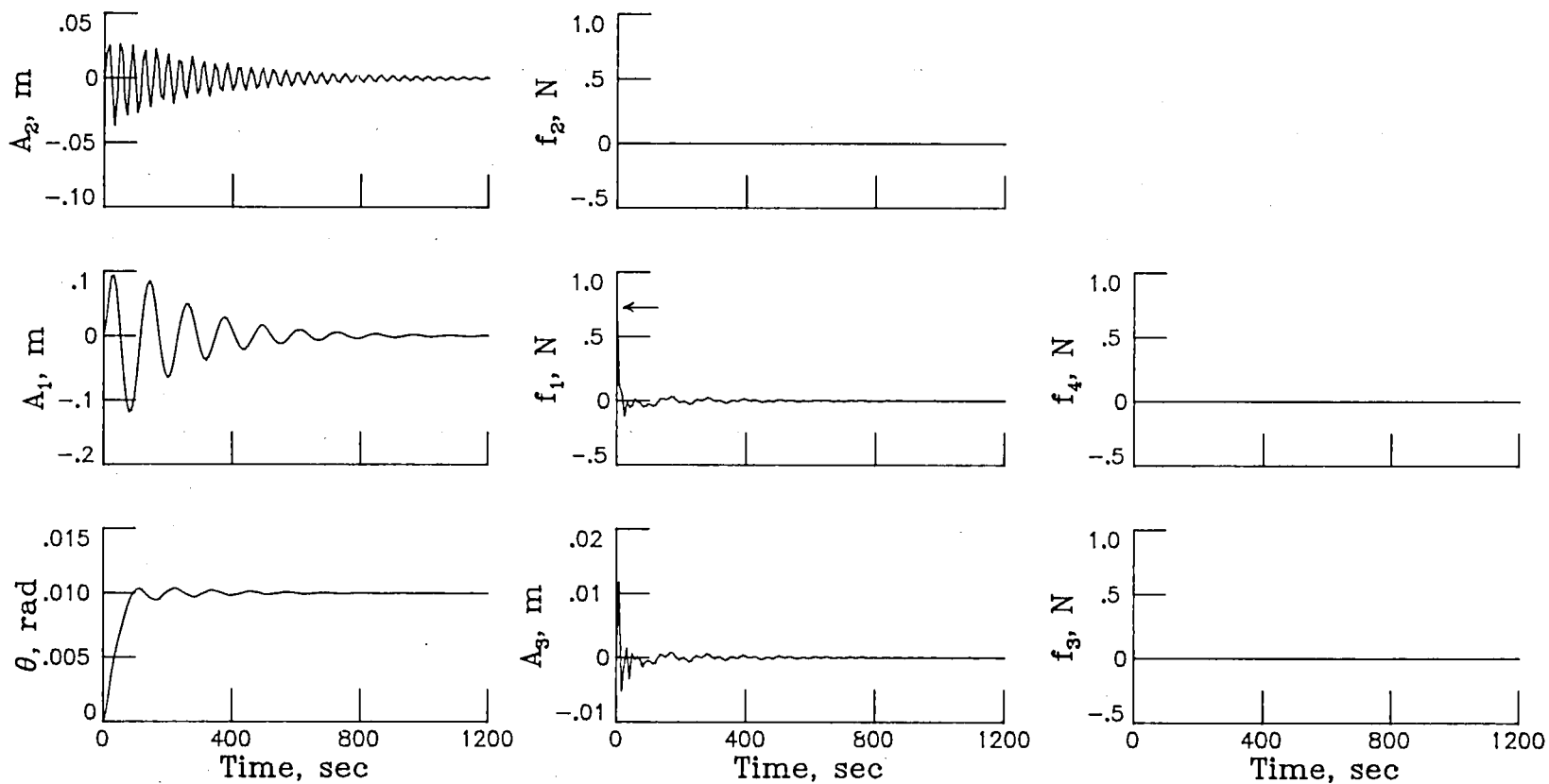
(e) Zero command; $F_{1,8}$ gain value readjusted.

Figure 6.- Concluded.



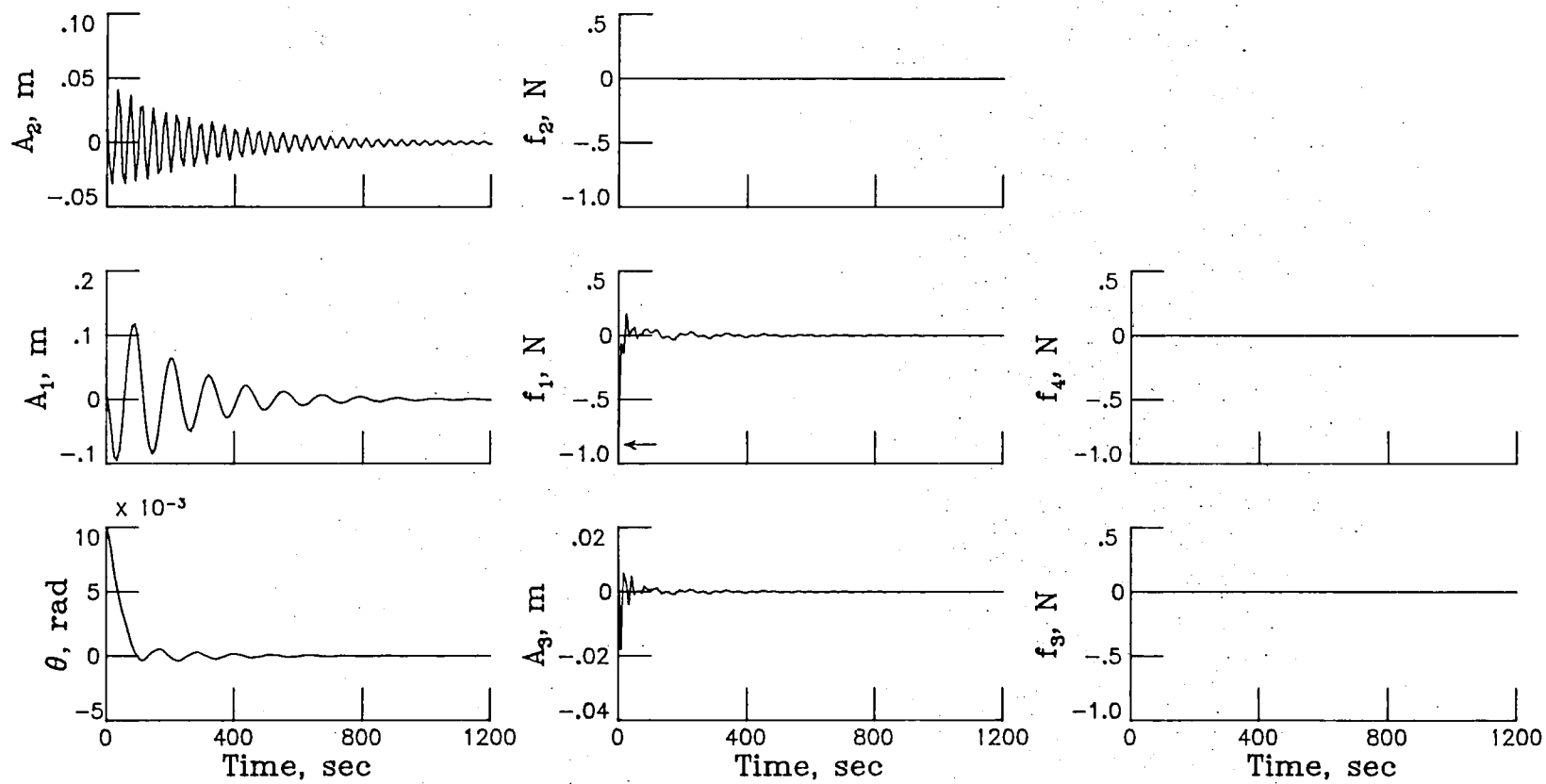
(a) Pitch command; feedback not adjusted.

Figure 7.- Response characteristics for completely decoupled control model with three control actuators eliminated. $\omega_\theta, \omega_1, \omega_2, \omega_3 = 0.1$ rad/sec; $\zeta_\theta, \zeta_1, \zeta_2, \zeta_3 = 0.5$. Arrows indicate initial values.



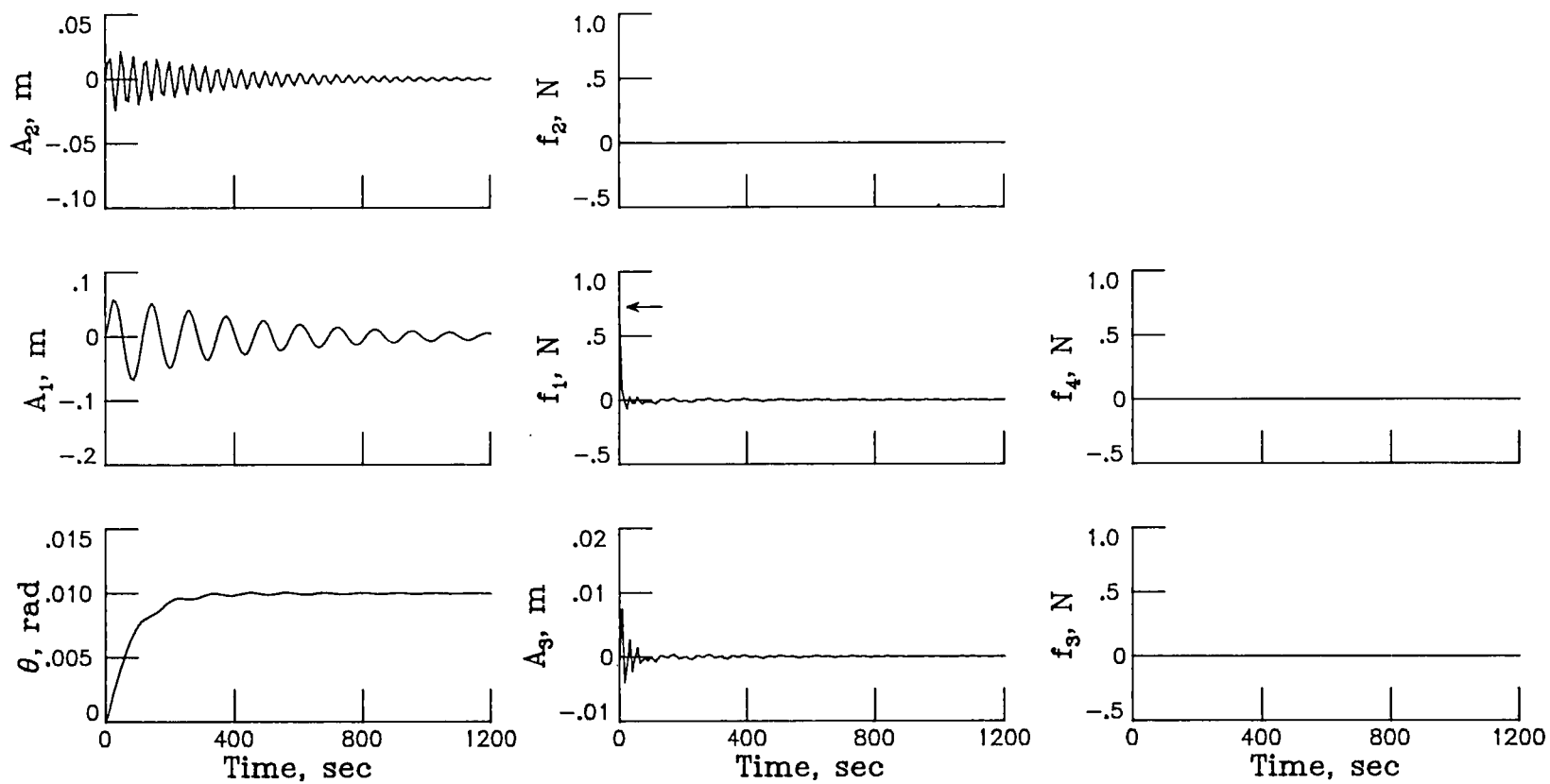
(b) Pitch command; feedback adjusted.

Figure 7.- Continued.



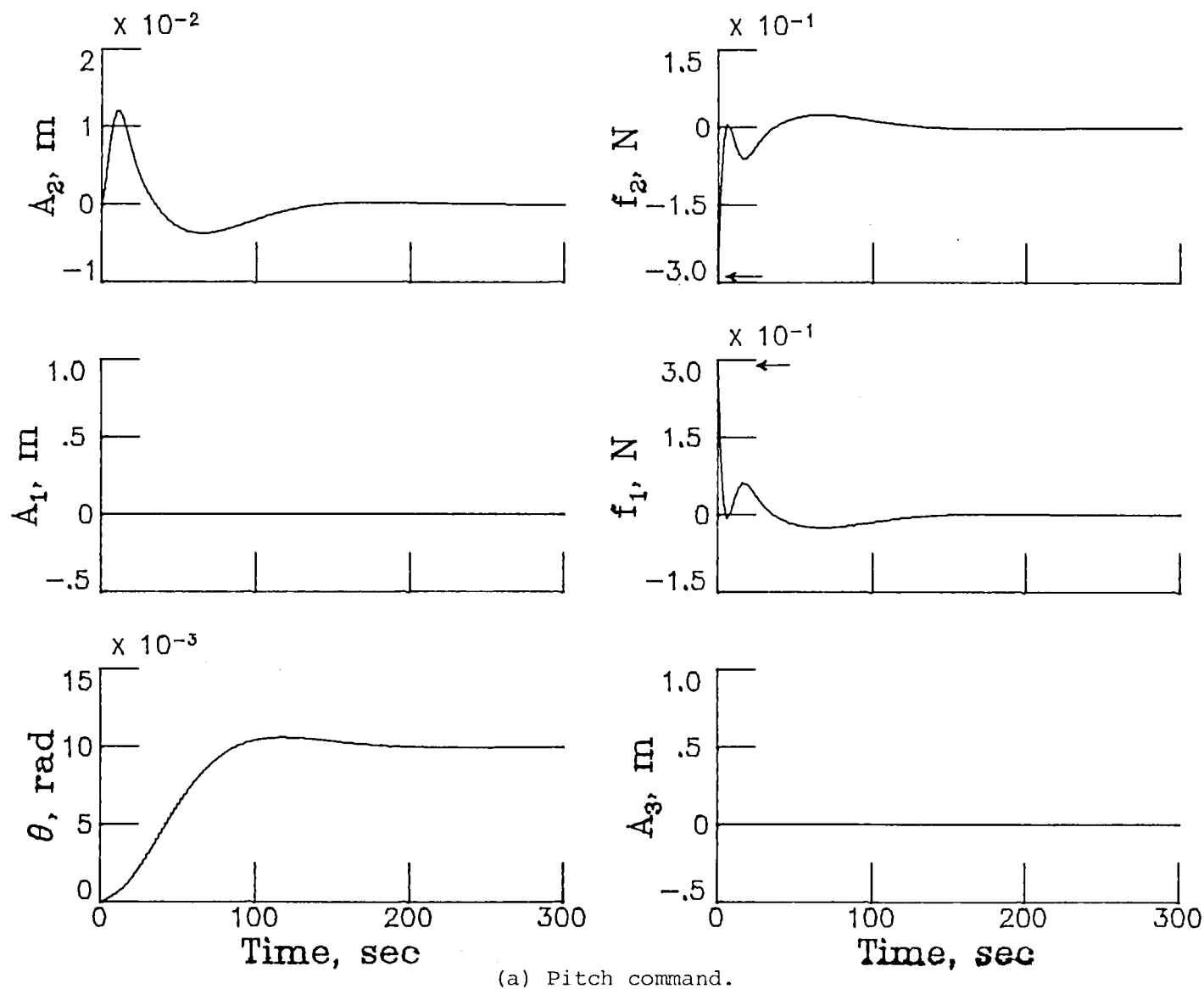
(c) Zero command; feedback adjusted.

Figure 7.- Continued.



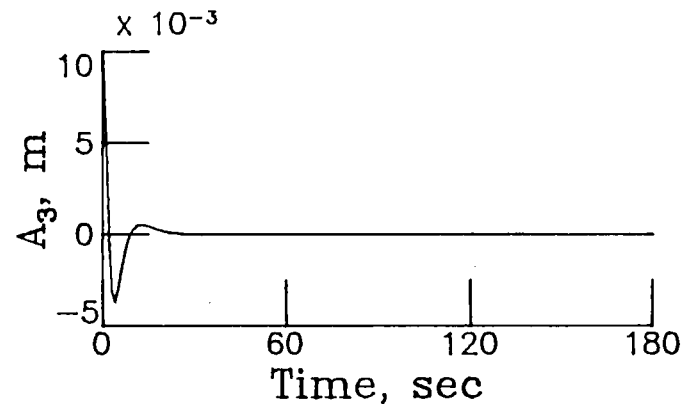
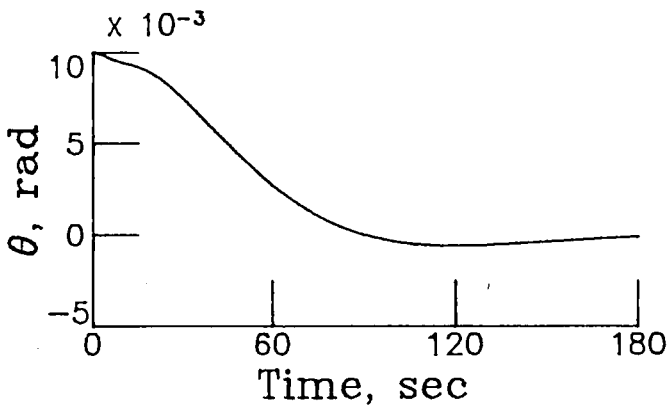
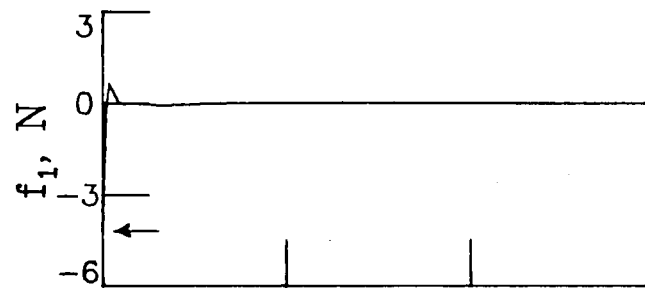
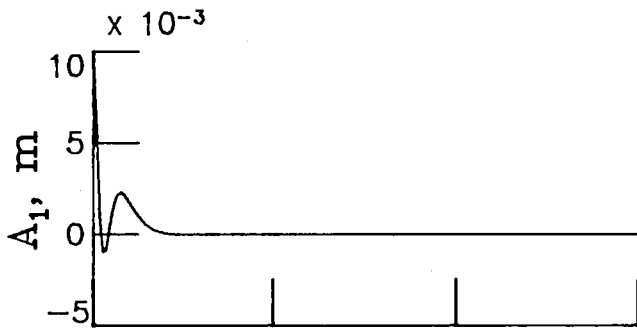
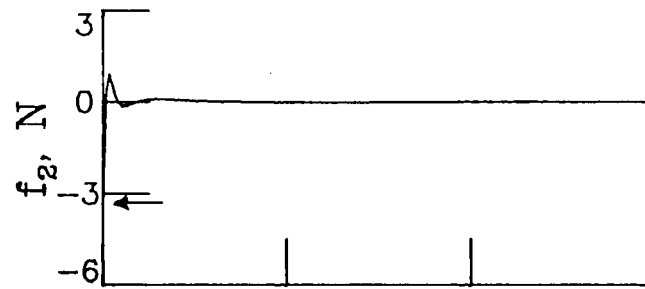
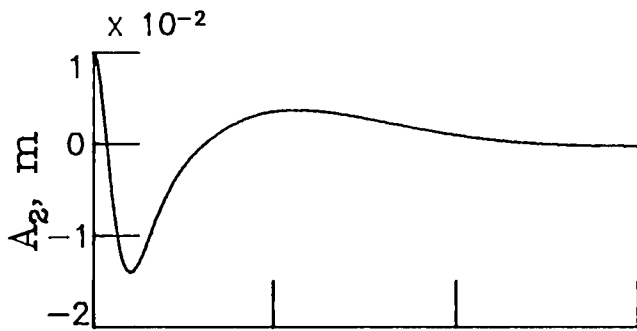
(d) Pitch command; feedback readjusted ($F_{1,5}$ to $F_{1,8}$ gain values doubled).

Figure 7.- Concluded.



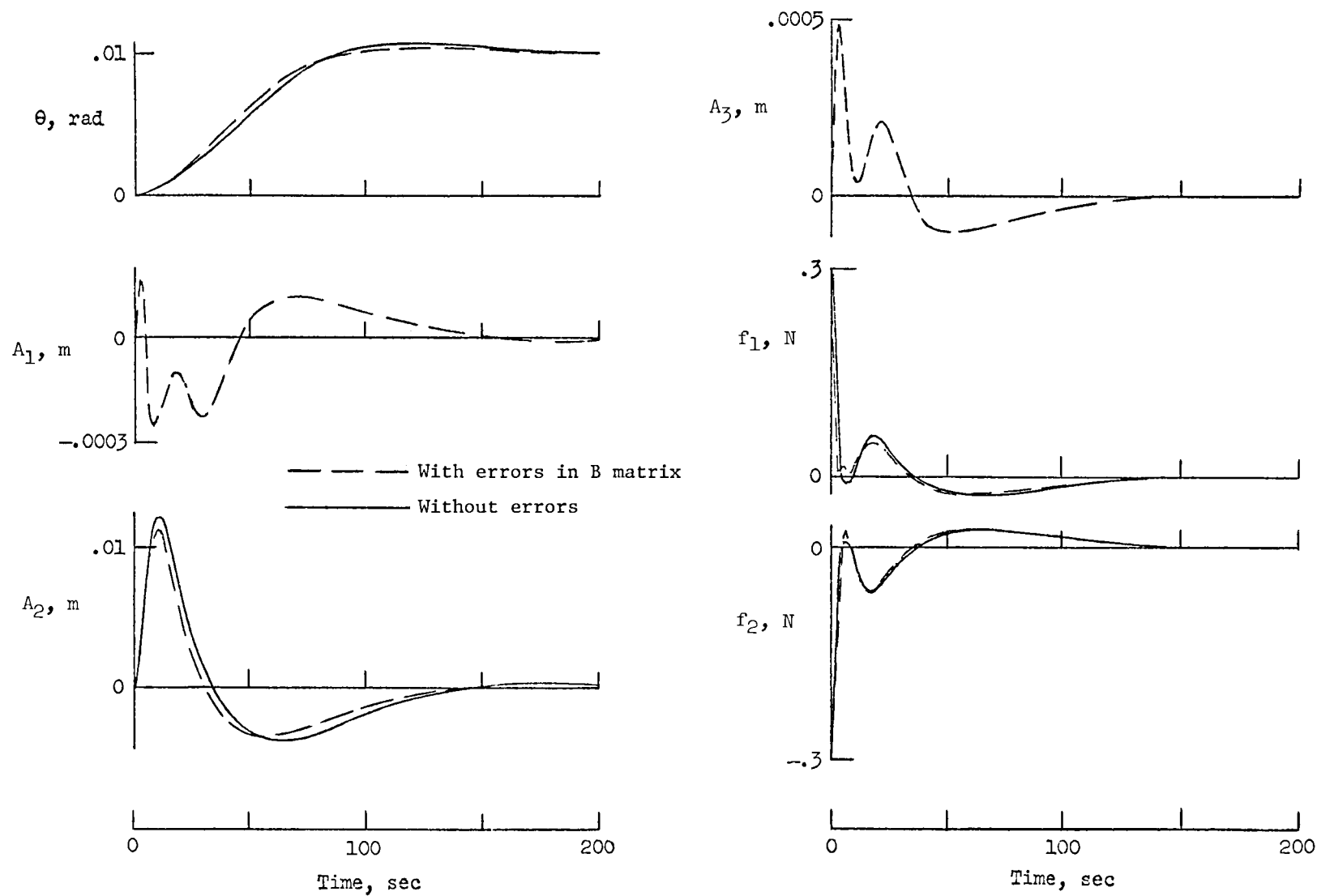
(a) Pitch command.

Figure 8.- Response characteristics for incompletely decoupled control model (one control actuator exactly at each end of beam). $\omega_0 = 0.1$ rad/sec; $\omega_1 = 0.5$ rad/sec; $\zeta_0, \zeta_1 = 1.0$. Arrows indicate initial values.



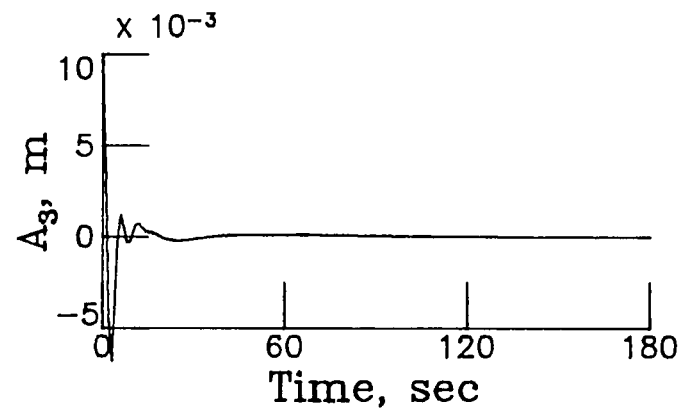
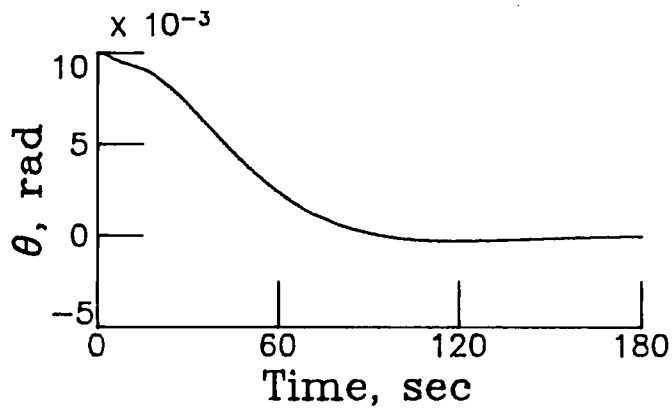
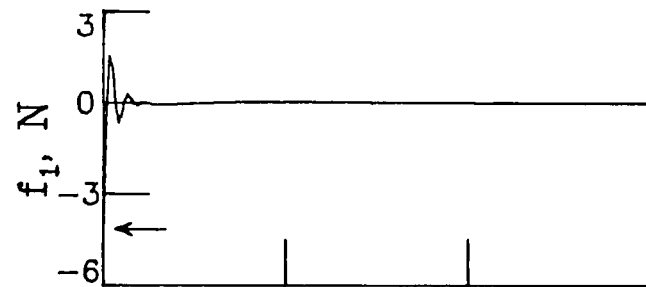
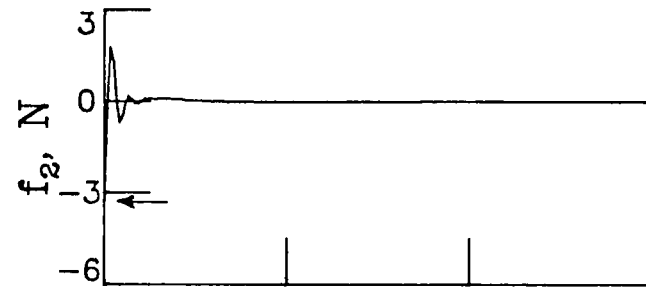
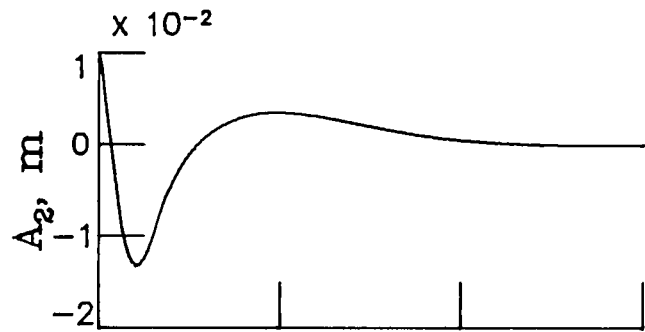
(b) Zero command.

Figure 8.- Continued.



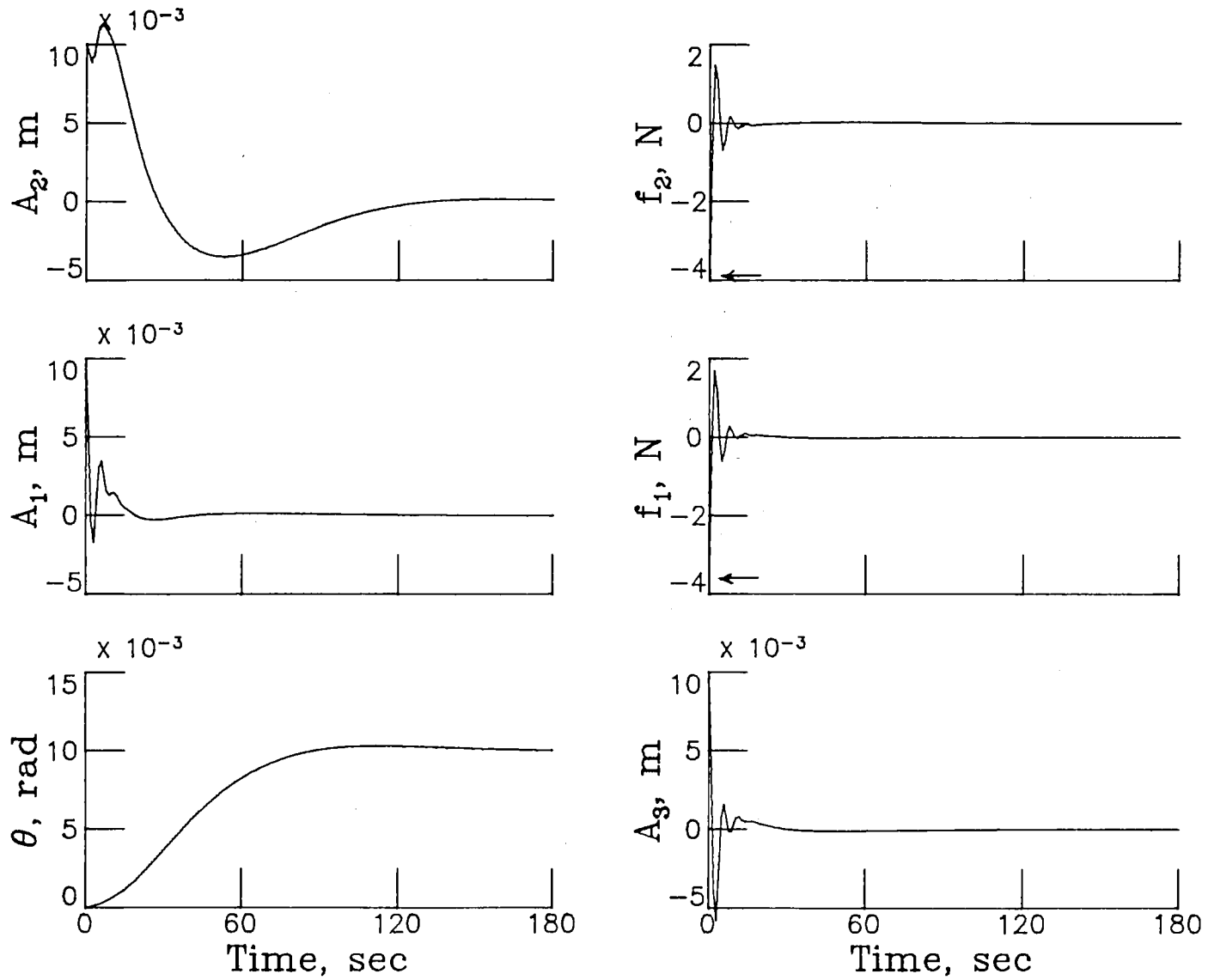
(c) Pitch command with parameter uncertainty.

Figure 8.- Continued.



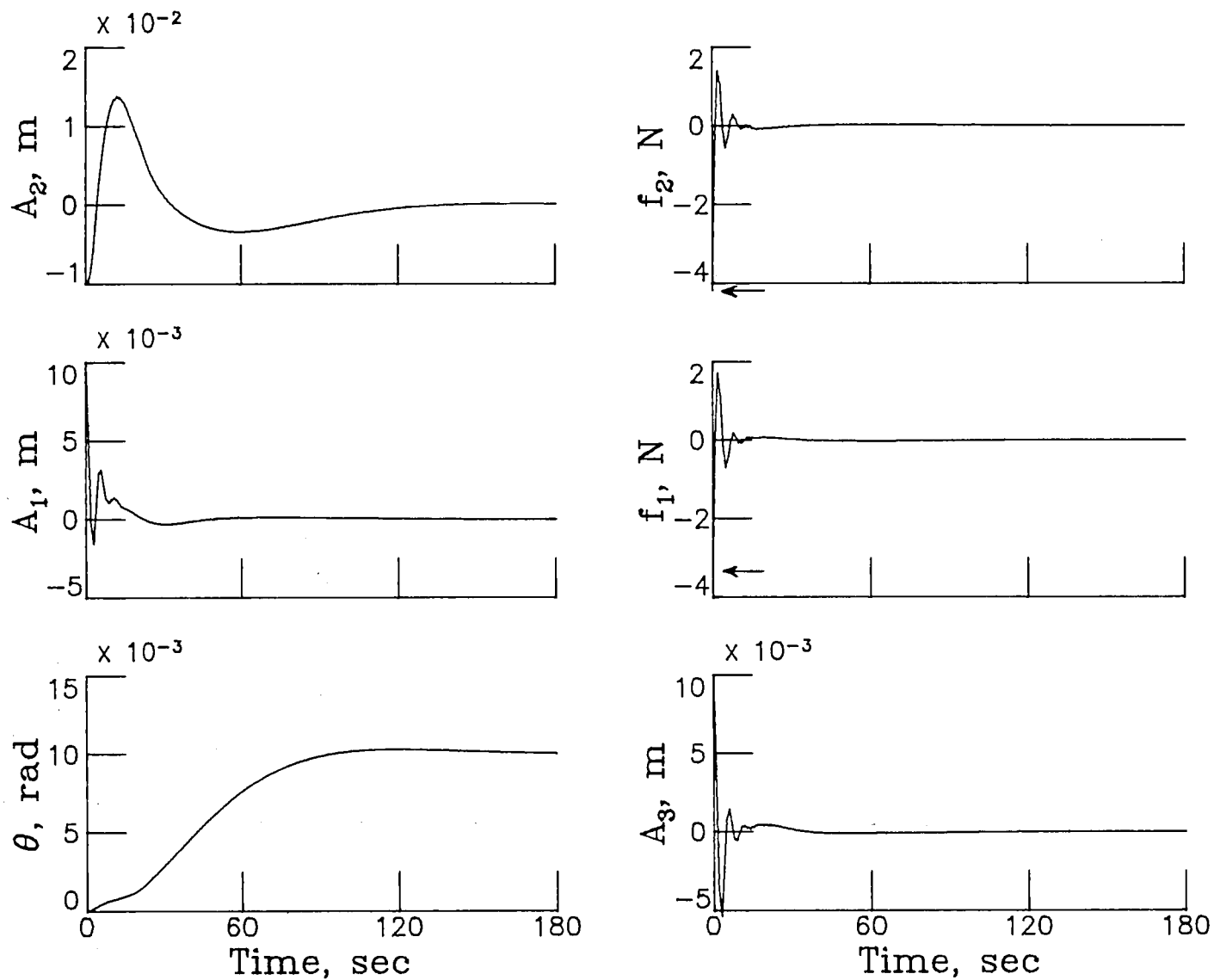
(d) Zero command with parameter uncertainty.

Figure 8.- Continued.



(e) Combined pitch command and flexible mode zero command with parameter uncertainty, initial A_2 positive.

Figure 8.- Continued.



(f) Combined pitch command and flexible mode zero command with parameter uncertainty, initial A_2 negative.

Figure 8.- Concluded.

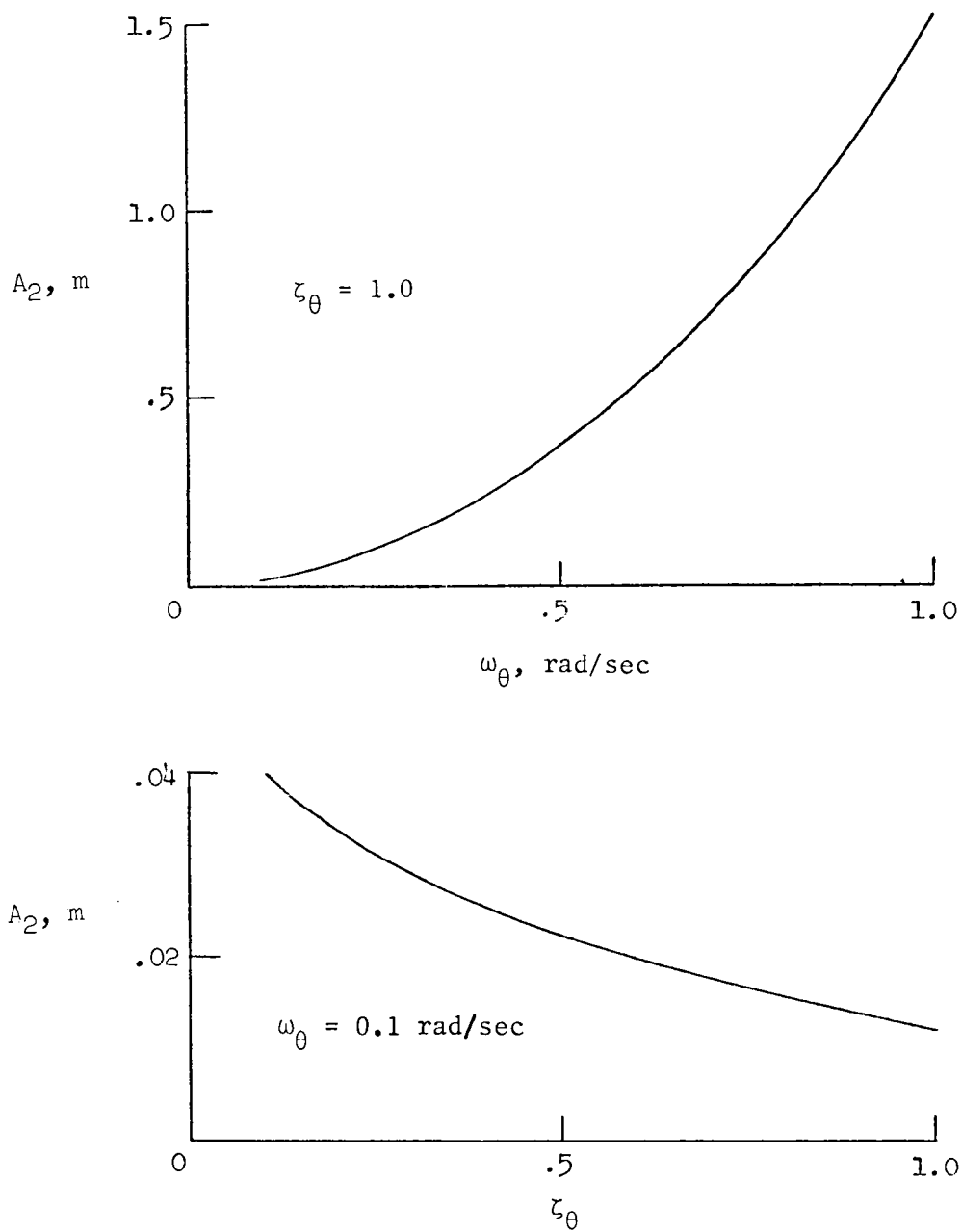


Figure 9.- Effect of closed-loop pitch dynamics on second flexible mode initial response for 0.01-rad pitch command. Pitch and first flexible mode decoupled with control actuators located at each end of beam.

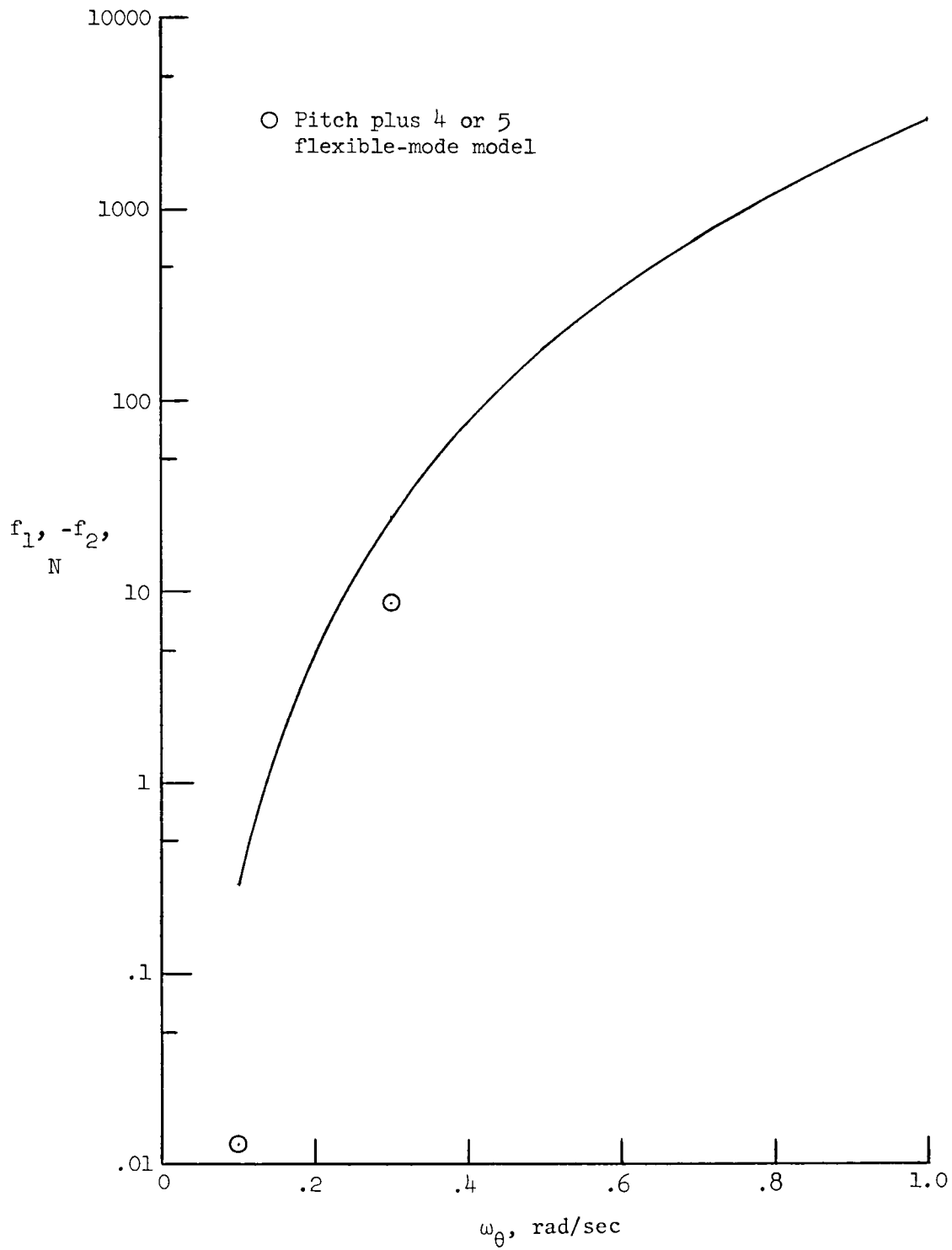


Figure 10.- Control-force requirements for 0.01-rad pitch command. Force is independent of all other closed-loop dynamics. (One control actuator located at each end of beam.)

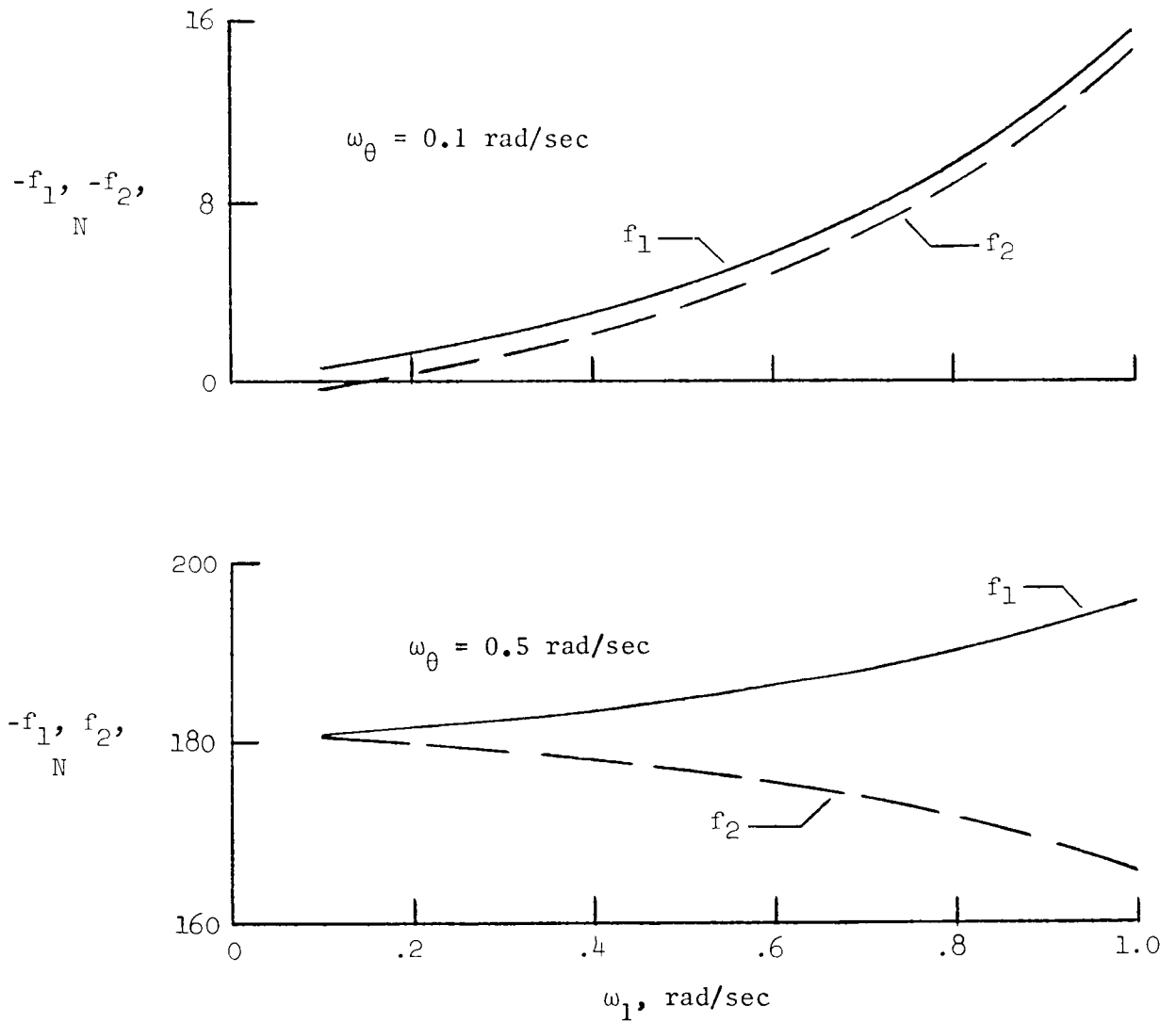
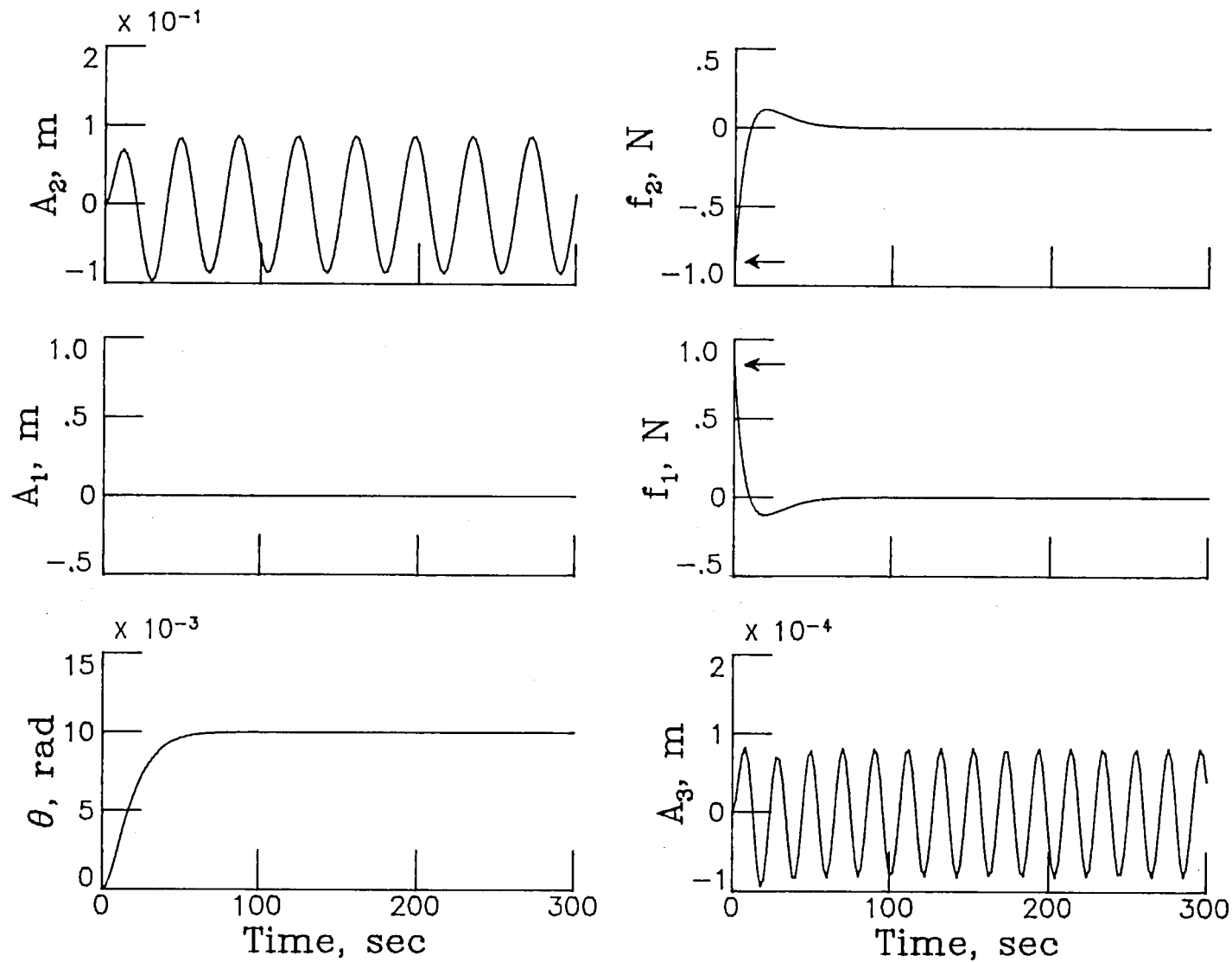
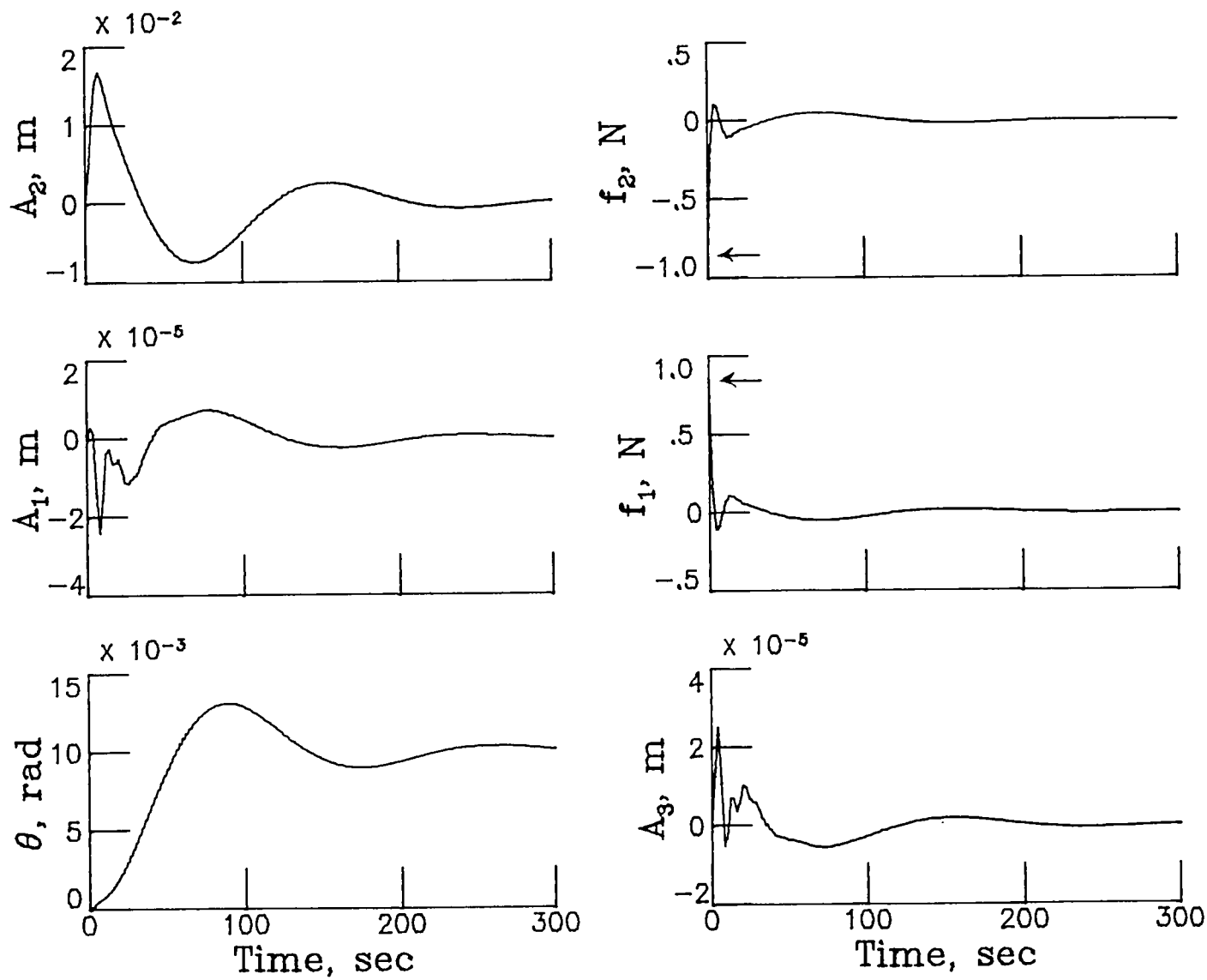


Figure 11.- Control-force requirements for zero command.
(Pitch plus three flexible mode model with one control
actuator at each end of beam.) $\zeta_\theta, \zeta_1 = 1.0$.

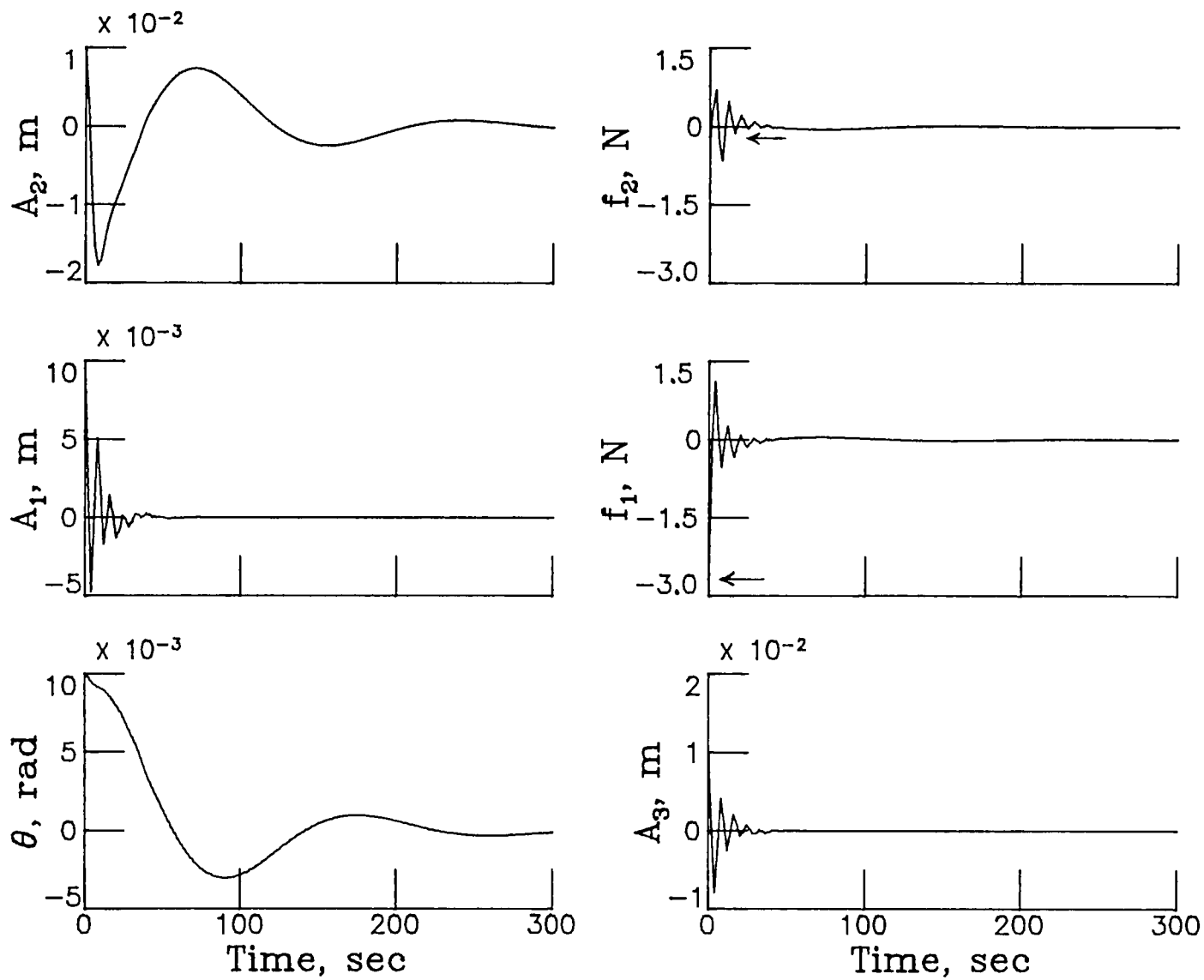


(a) Pitch command; feedback not adjusted.

Figure 12.- Response characteristics for incompletely decoupled control model (one control actuator at each end of beam, one slightly offset). $\omega_0 = 0.1$ rad/sec; $\omega_1 = 0.5$ rad/sec; $\zeta_0, \zeta_1 = 1.0$. Arrows indicate initial values.

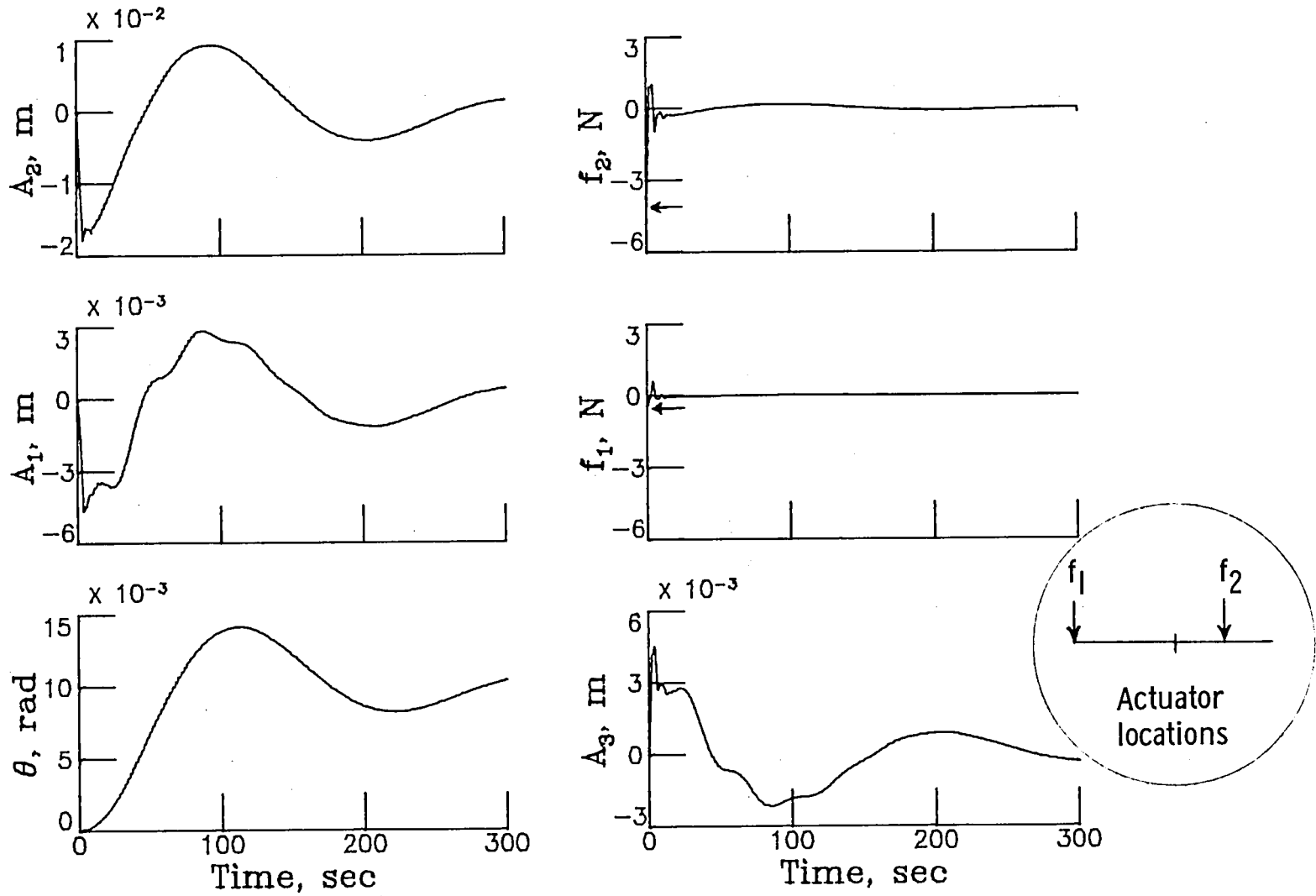


(b) Pitch command; feedback adjusted by ratio method.



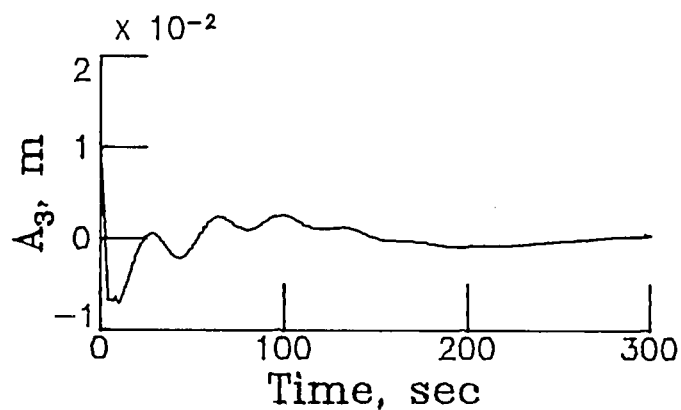
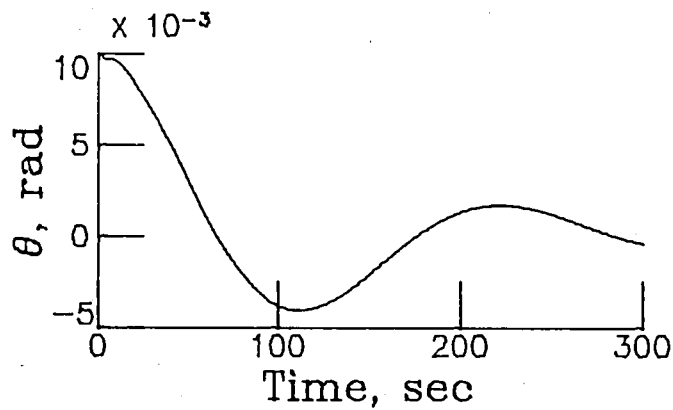
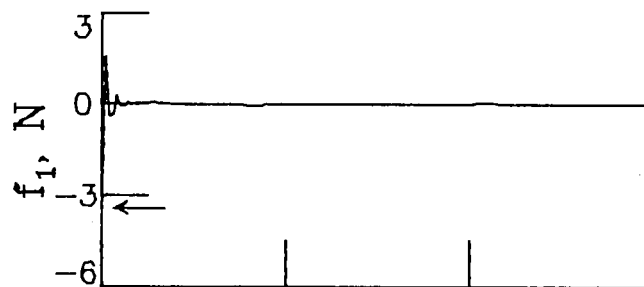
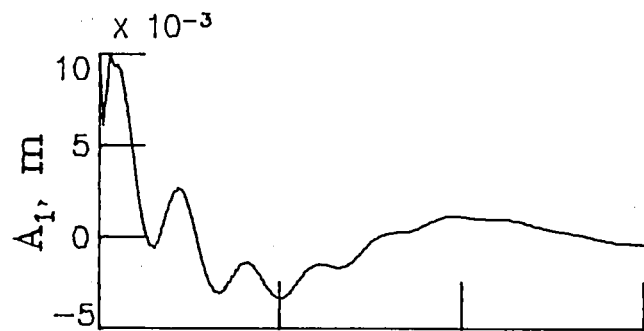
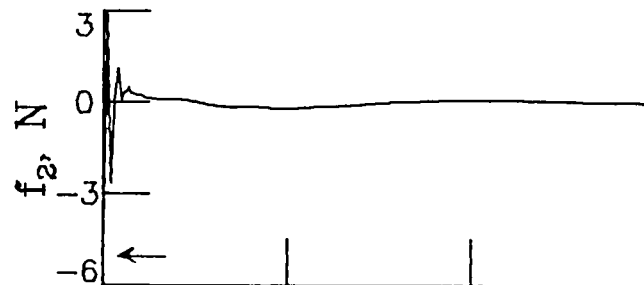
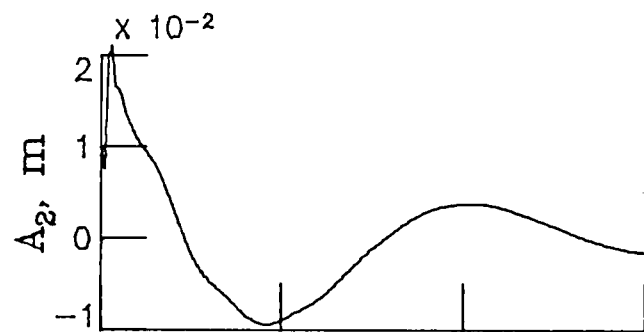
(c) Zero command; feedback adjusted by ratio method.

Figure 12.- Concluded.



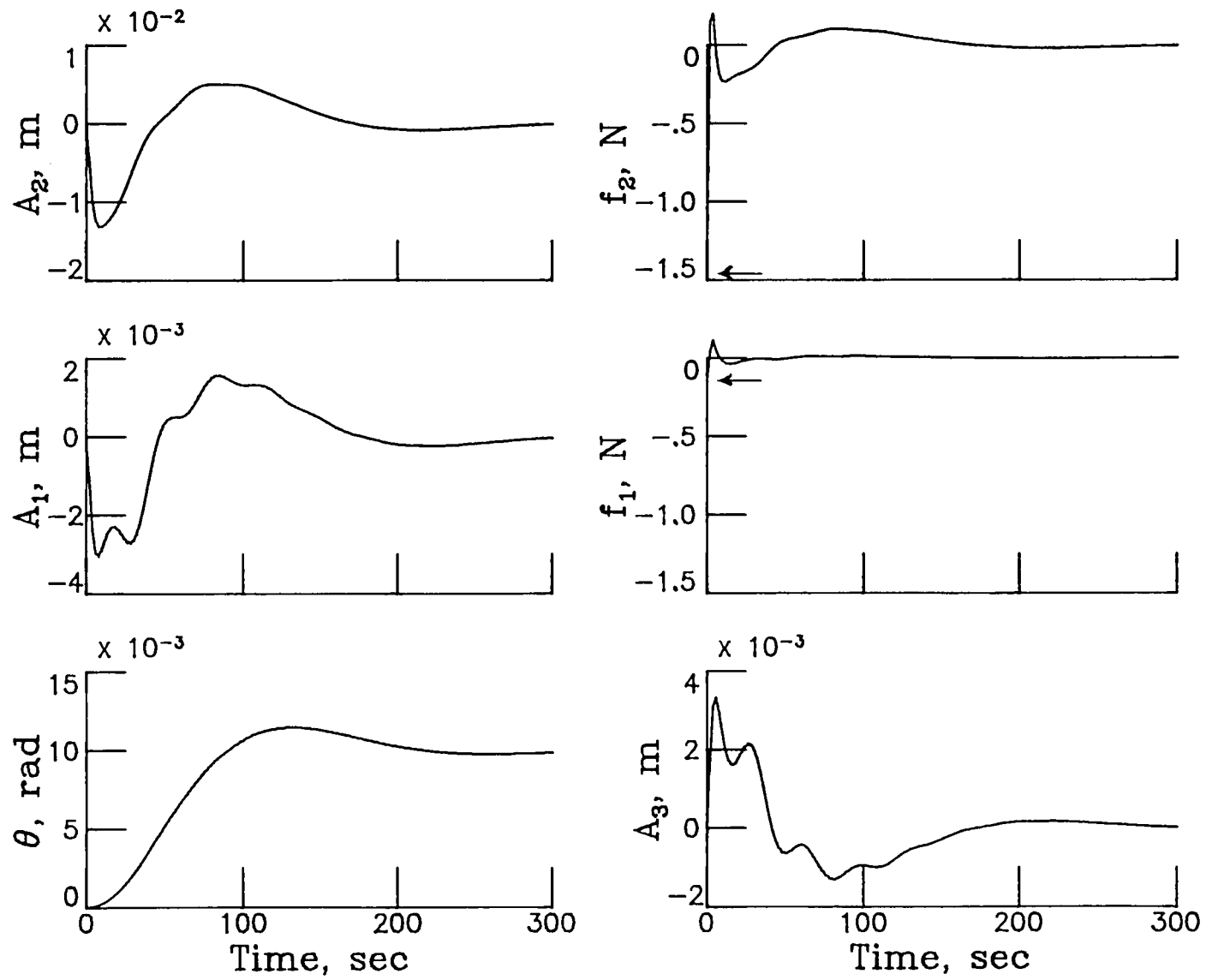
(a) Pitch command; feedback adjusted by ratio method.

Figure 13.- Response characteristics for incompletely decoupled control model. $\omega_\theta = 0.1$ rad/sec; $\omega_1 = 0.5$ rad/sec; $\zeta_\theta, \zeta_1 = 1.0$. Arrows indicate initial values.

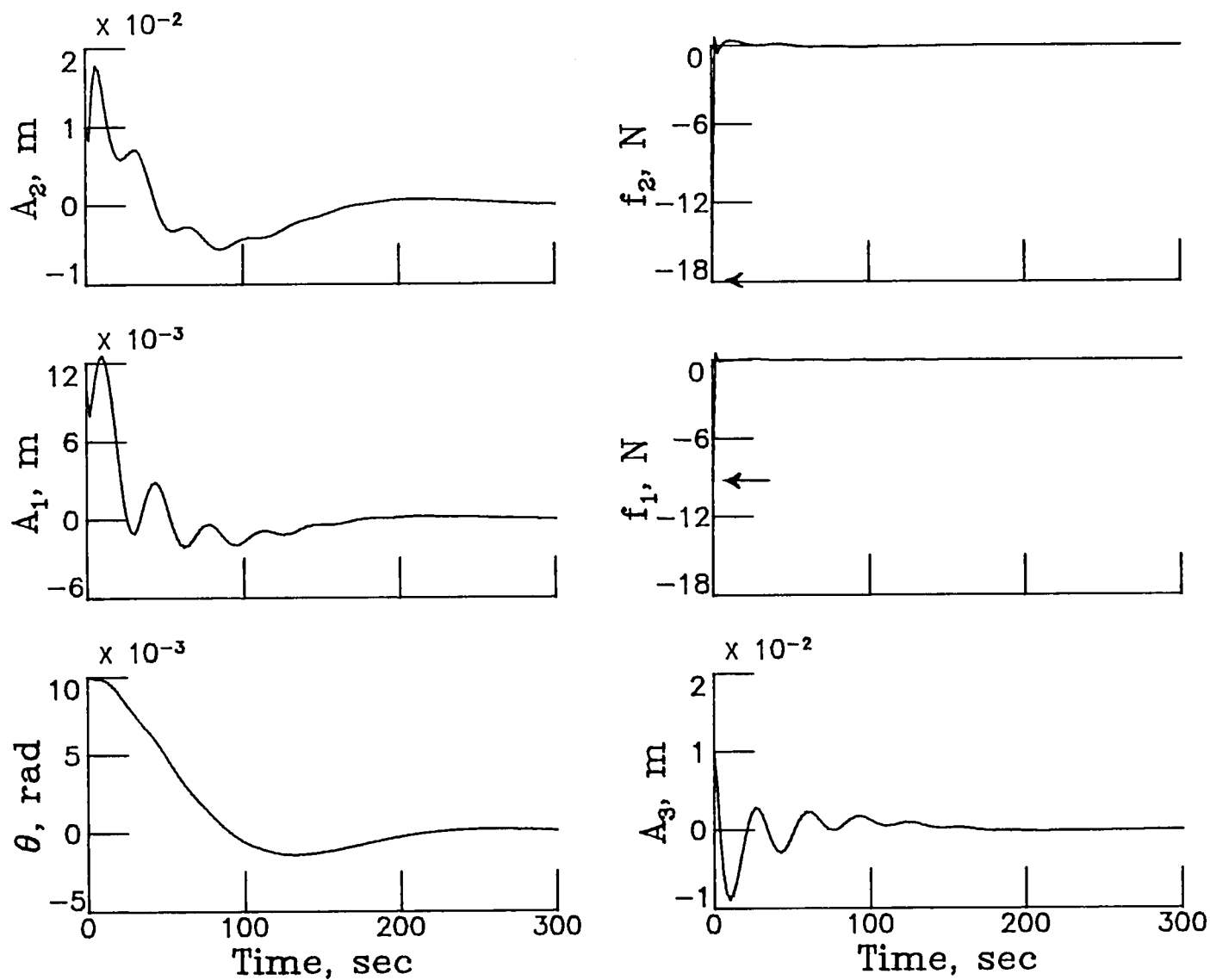


(b) Zero command; feedback adjusted by ratio method.

Figure 13.- Continued.

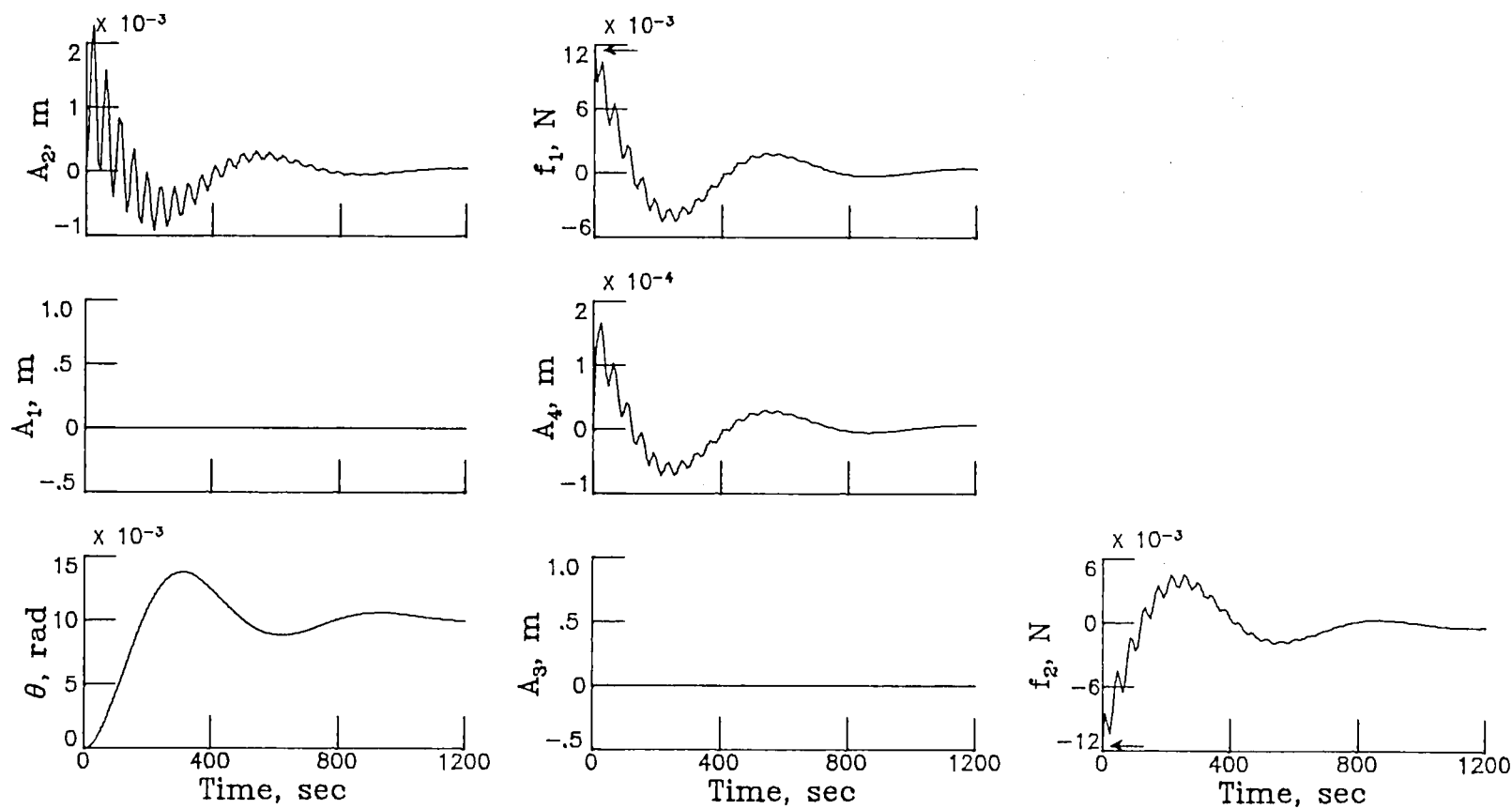


(c) Pitch command; feedback adjusted by alternate method.



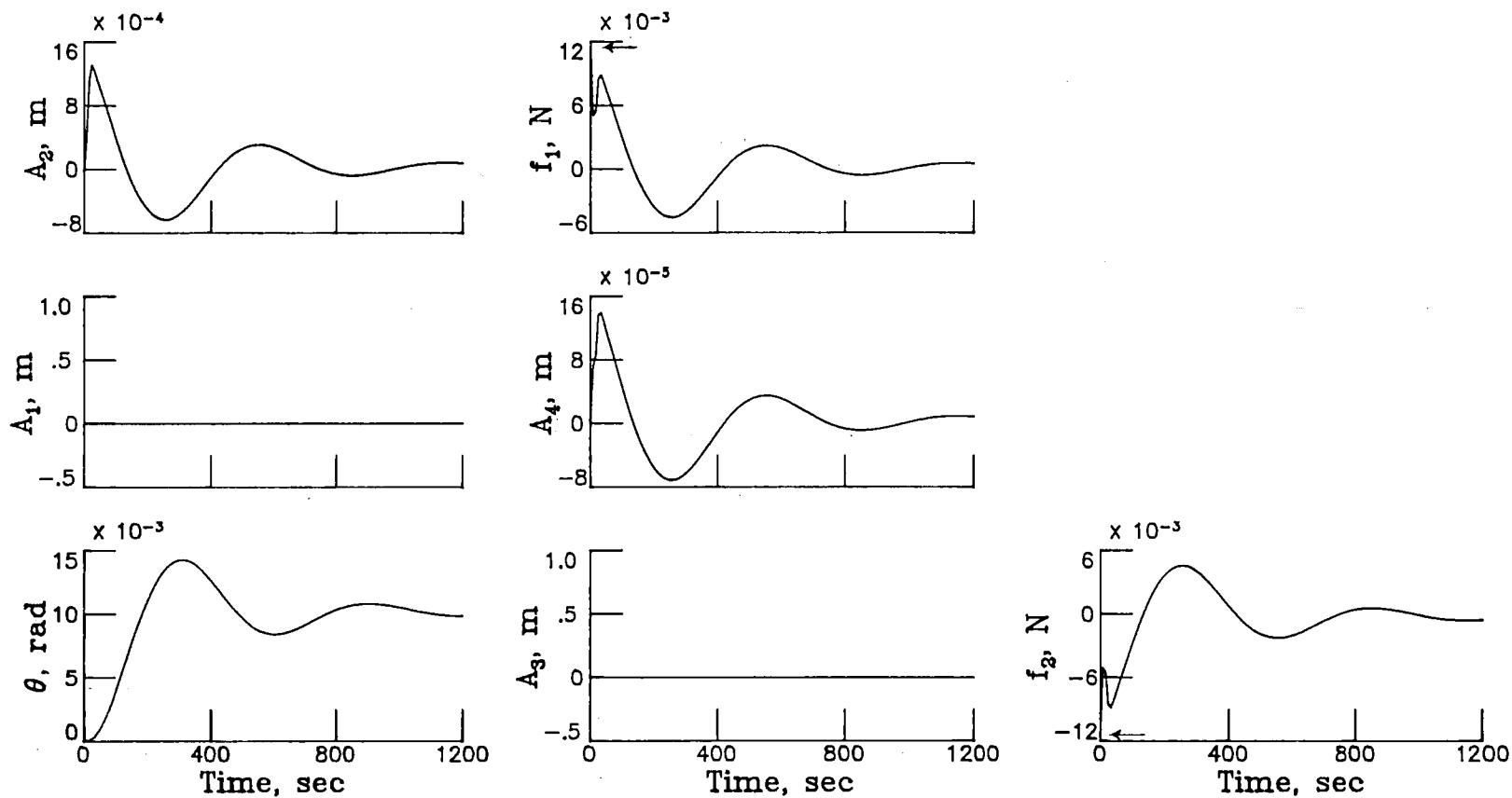
(d) Zero command; feedback adjusted by alternate method.

Figure 13.- Concluded.



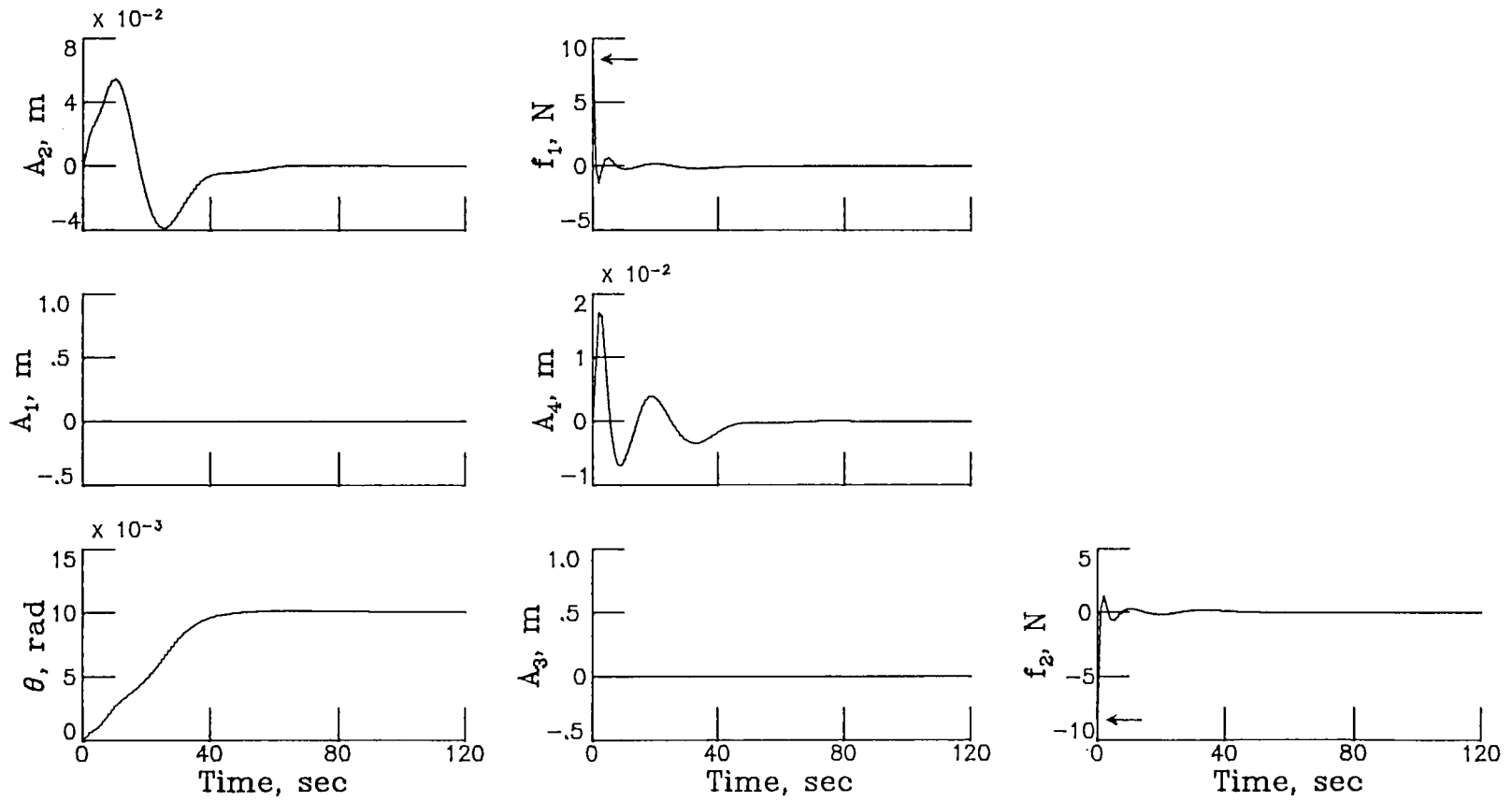
(a) Pitch command.

Figure 14.- Response characteristics for incompletely decoupled control model. (One control actuator at each end of beam.) $\omega_\theta = 0.1$ rad/sec; $\omega_1 = 0.5$ rad/sec; $\zeta_\theta, \zeta_1 = 1.0$. Arrows indicate initial values.



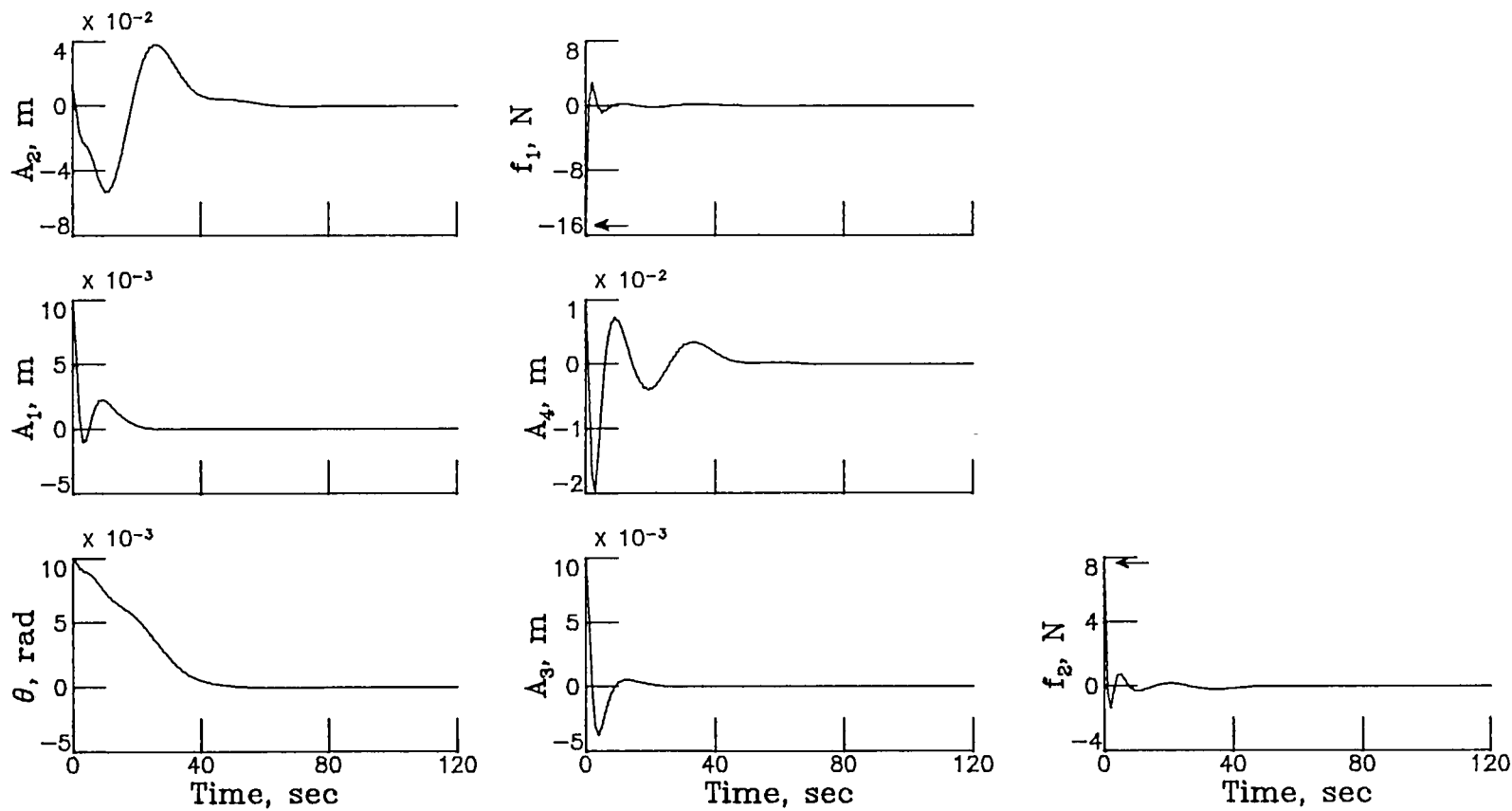
(b) Pitch command; feedback adjusted.

Figure 14.- Continued.



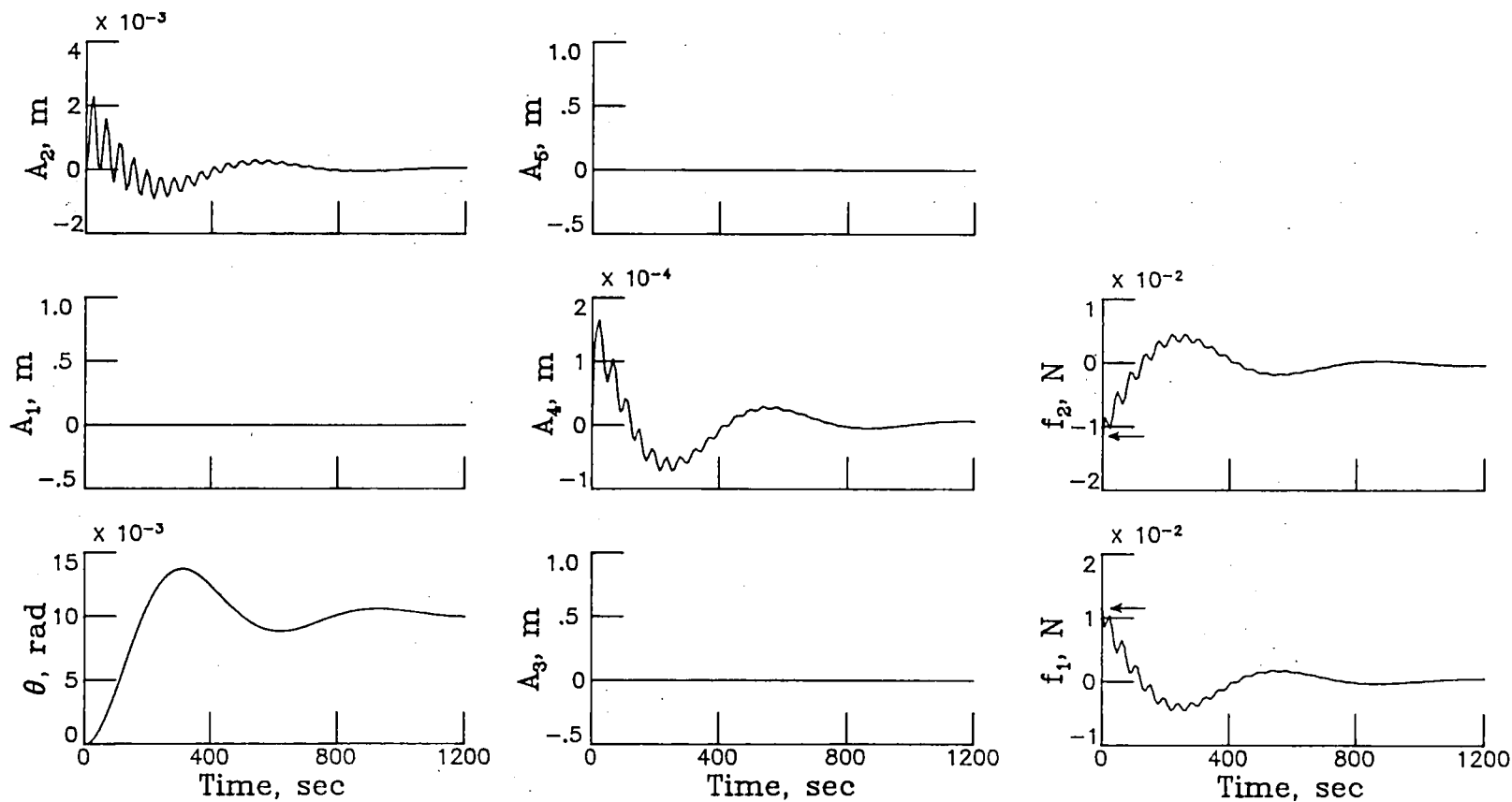
(c) Pitch command with ω_0 changed to 0.3 rad/sec.

Figure 14.- Continued.



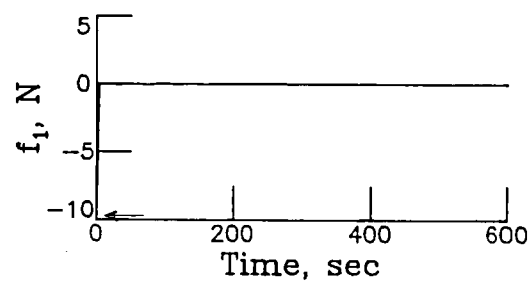
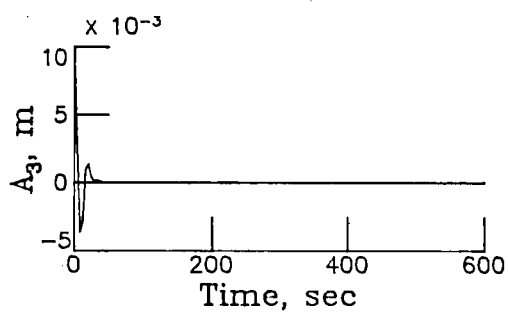
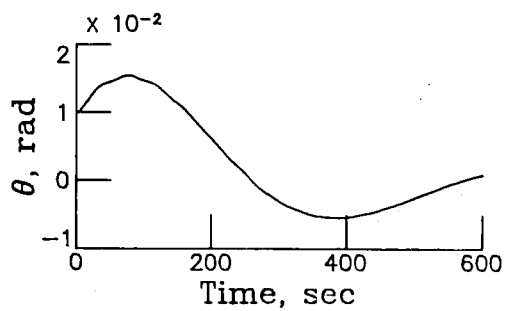
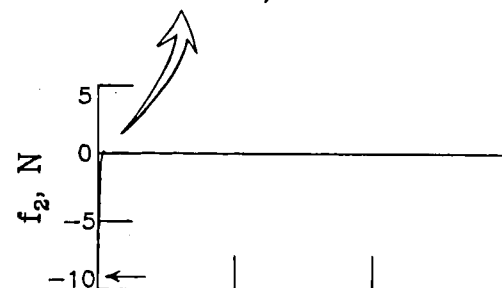
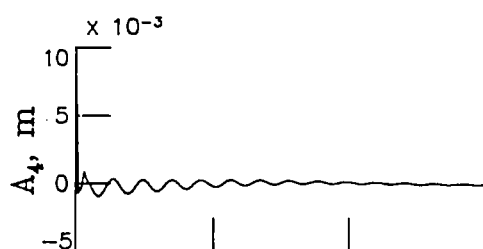
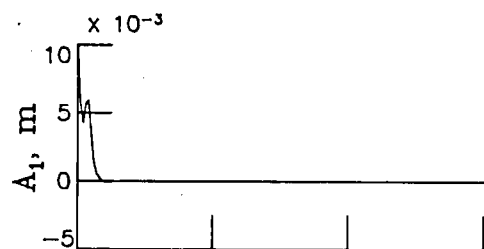
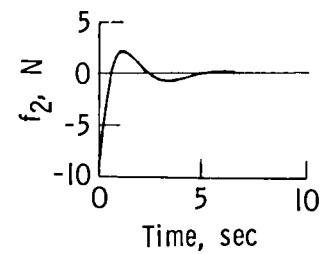
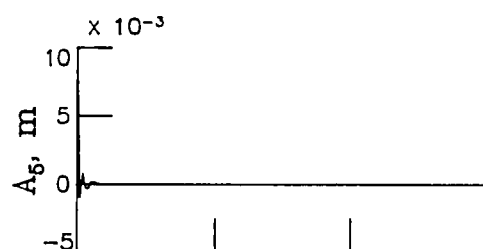
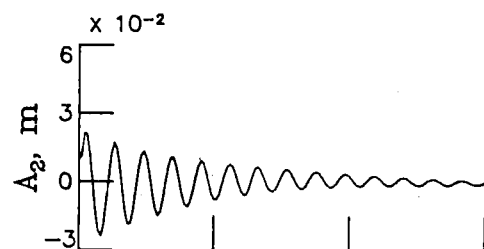
(d) Zero command with ω_θ changed to 0.3 rad/sec.

Figure 14.- Concluded.



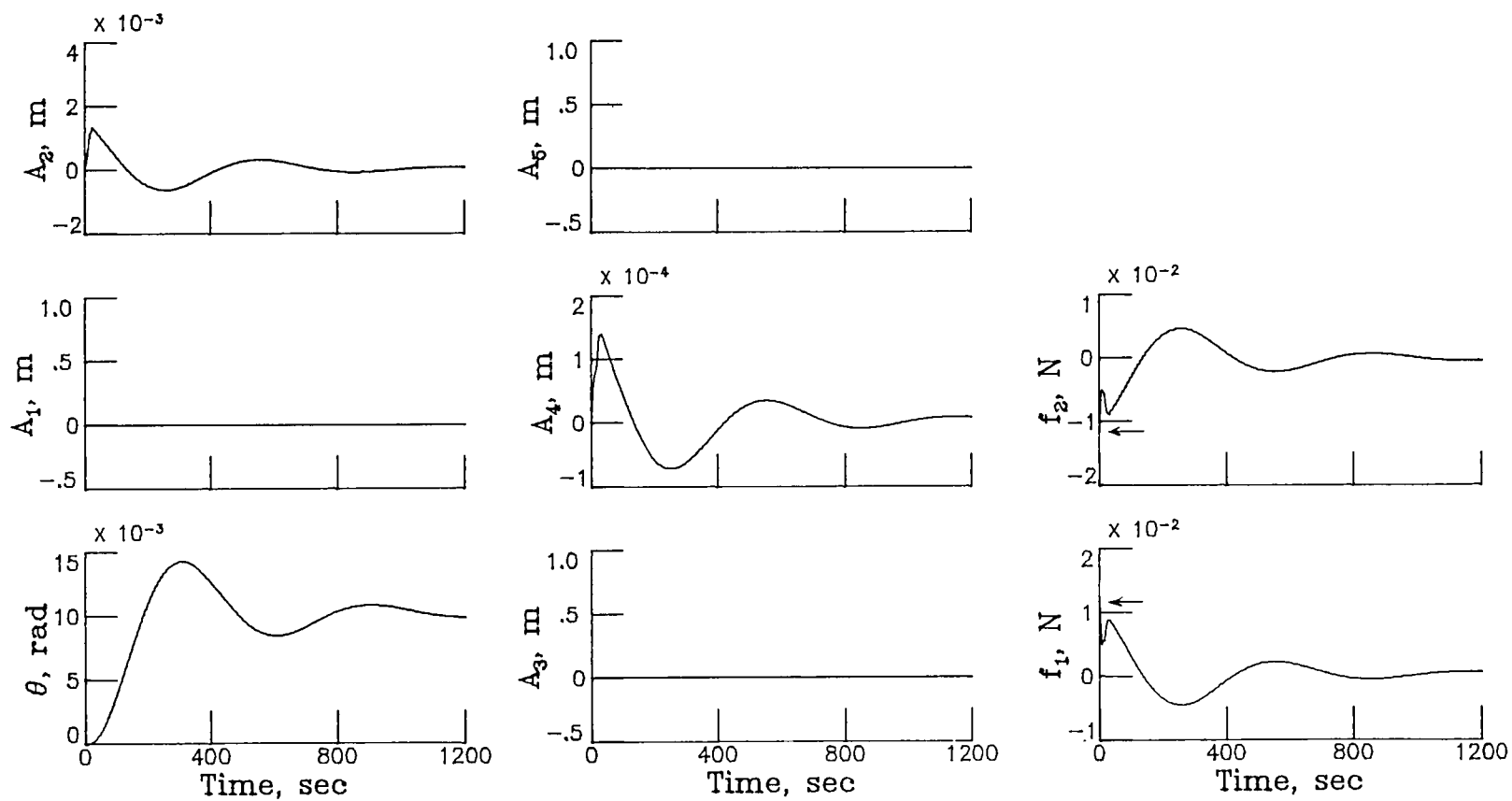
(a) Pitch command.

Figure 15.- Response characteristics for incompletely decoupled control model. (One control actuator at each end of beam.) $\omega_0 = 0.1$ rad/sec; $\omega_1 = 0.5$ rad/sec; $\zeta_0, \zeta_1 = 1.0$. Arrows indicate initial values.



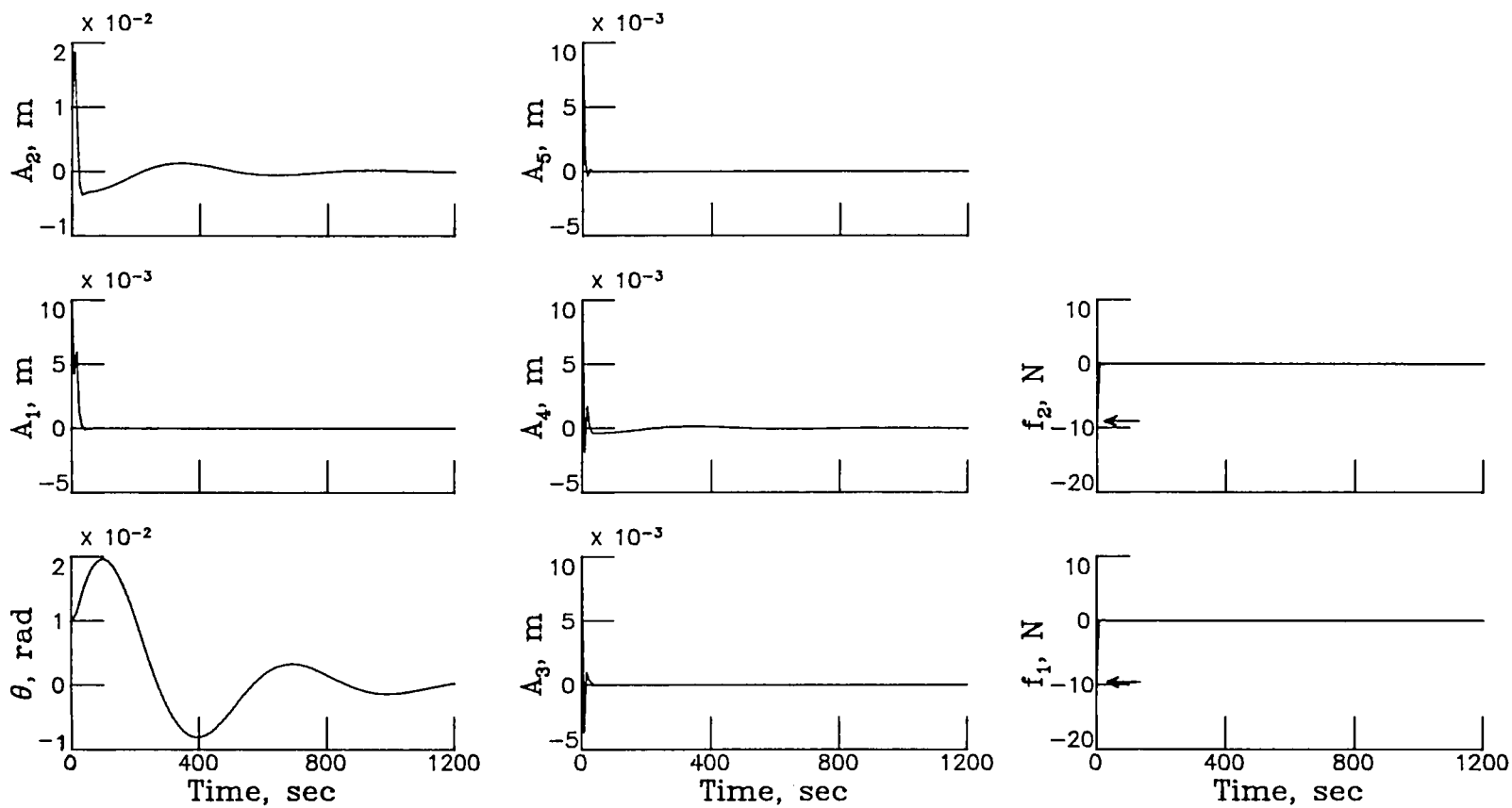
(b) Zero command.

Figure 15.- Continued.



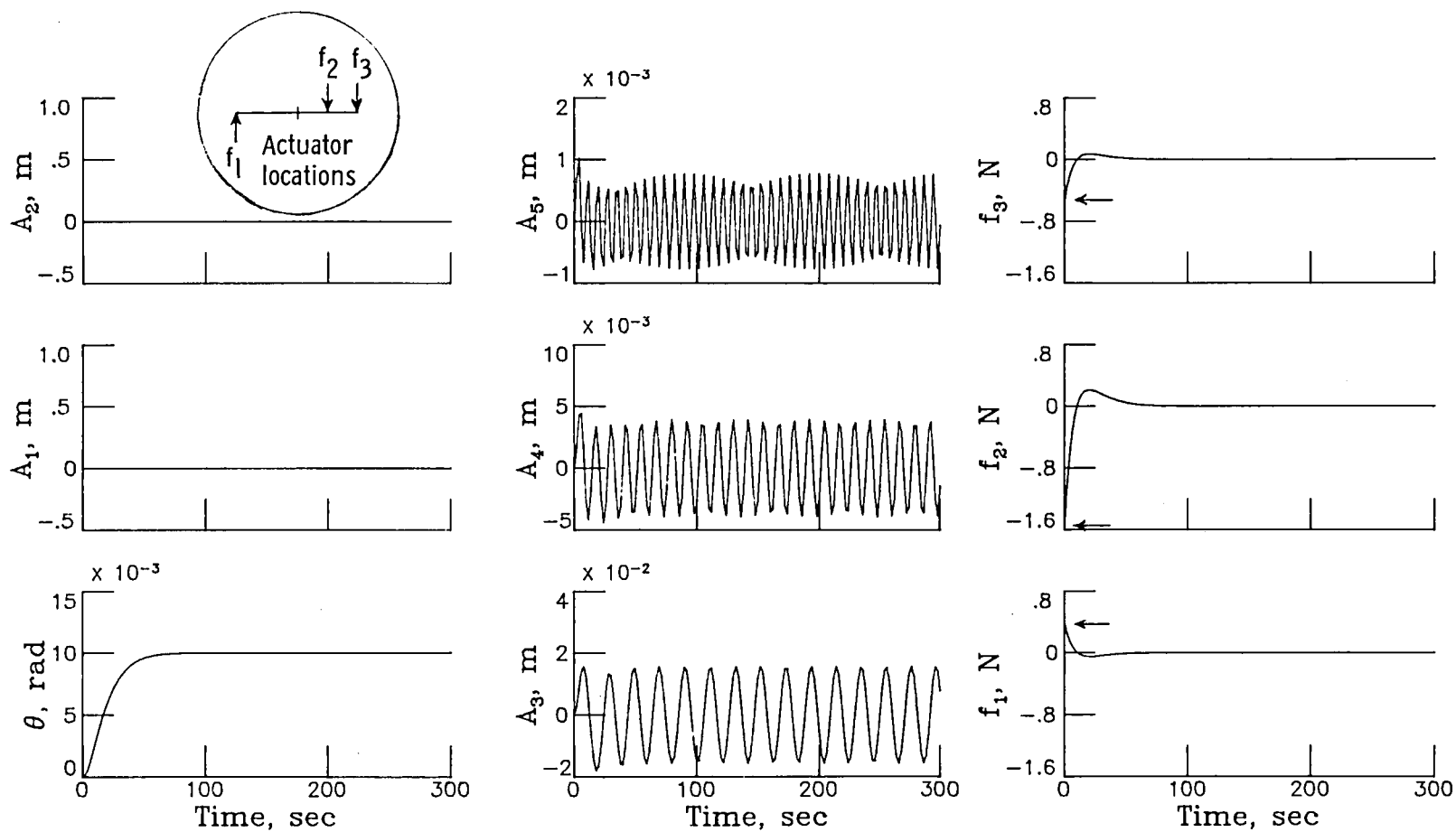
(c) Pitch command; feedback adjusted.

Figure 15.- Continued.



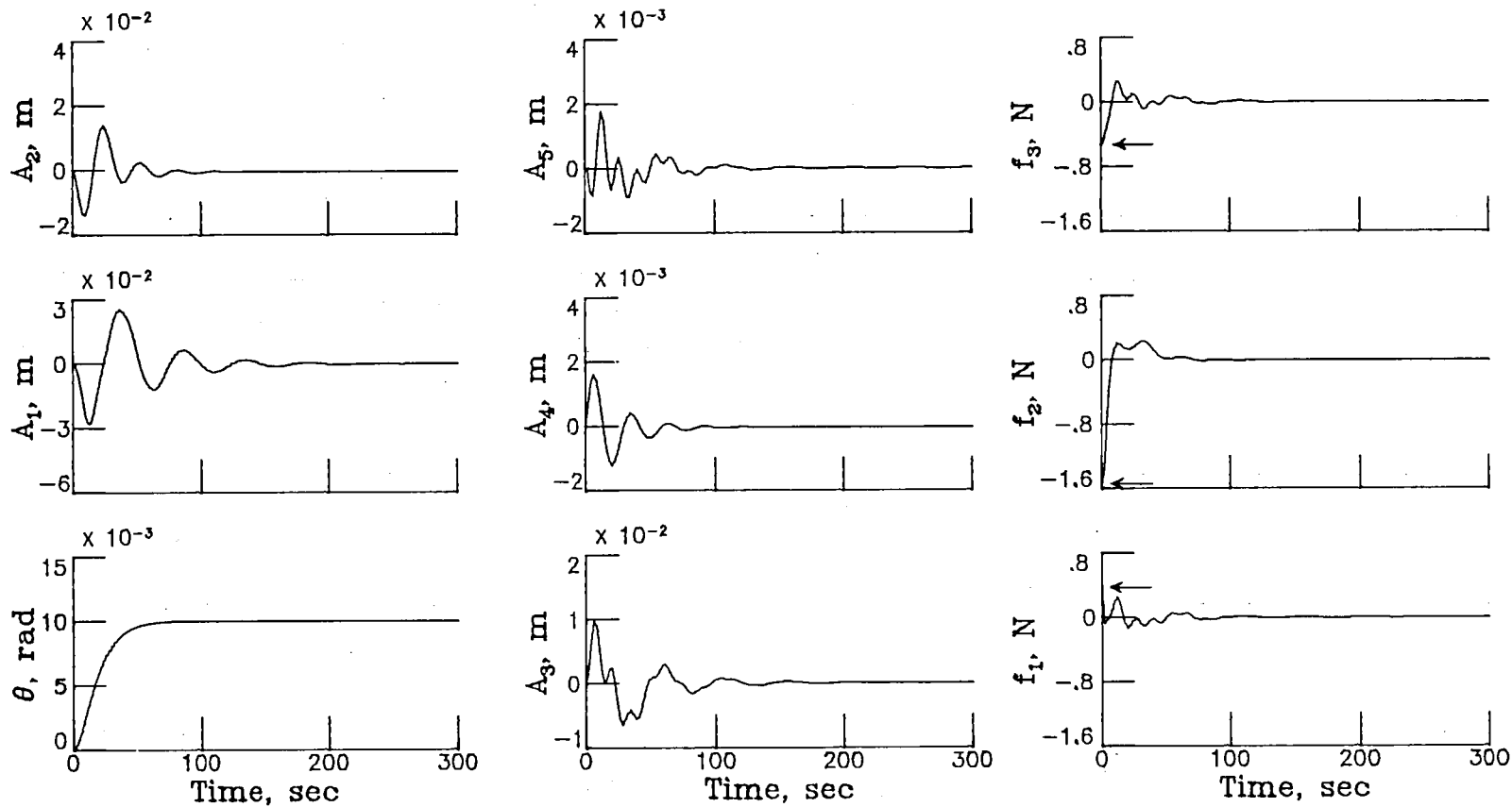
(d) Zero command; feedback adjusted.

Figure 15.- Concluded.



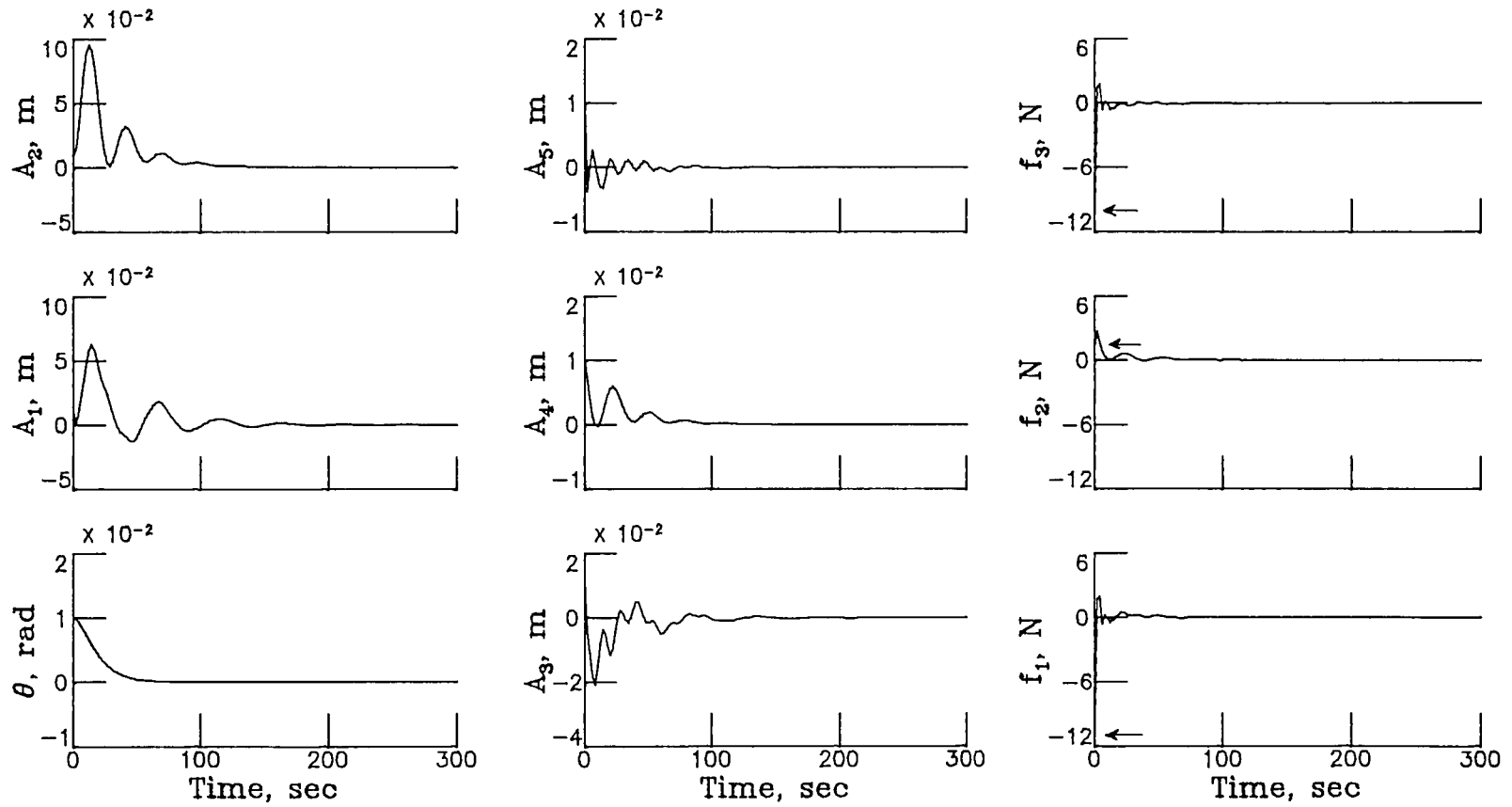
(a) Pitch command; feedback not adjusted.

Figure 16.- Response characteristics for incompletely decoupled control model. (Three control actuators used.) $\omega_\theta = 0.1$ rad/sec; $\omega_1, \omega_2 = 0.5$ rad/sec; $\zeta_\theta, \zeta_1, \zeta_2 = 1.0$. Arrows indicate initial values.



(b) Pitch command with ω_2 changed to 0.1 rad/sec;
feedback adjusted by ratio method.

Figure 16.- Continued.



(c) Zero command with ω_2 changed to 0.1 rad/sec;
feedback adjusted by ratio method.

Figure 16.- Concluded.

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16. Abstract A study to develop procedures for applying decoupling theory in the control of a long, flexible beam in low Earth orbit was performed. Control involved commanding changes in pitch attitude as well as nulling initial disturbances in the pitch and flexible modes. Control-force requirements were analyzed. Also, the effects of parameter uncertainties on the decoupling process were analyzed and were found to be small. Two methods were investigated: (1) the system was completely decoupled (i.e., the number of control actuators equaled the number of modes in the model) and certain actuators were then eliminated, one by one, which resulted in some or all modes not fully controlled; (2) specified modes of the system were excluded from the decoupling control law by employing fewer control actuators than modes in the model. In both methods, adjustments were made in the feedback gains to include the uncontrolled modes in the overall control of the system.					
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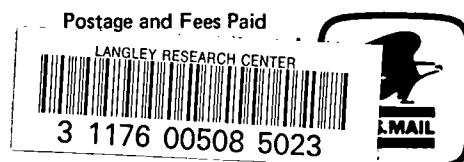
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